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ON POSITIVE WEAK SOLUTIONS FOR A CLASS OF WEIGHTED (P,Q)-LAPLACIAN NONLINEAR SYSTEM

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Abstract. In this paper we study the existence and nonexistence of positive weak solutions for the nonlinear system

$$\left.\begin{array}{rll} -\Delta_{P,p}u = \lambda a(x)v^{\gamma} + b(x)u^{\alpha} & \text{in} & \Omega, \\ -\Delta_{Q,q}v = \lambda c(x)u^{\delta} + d(x)v^{\beta} & \text{in} & \Omega, \\ u = v = 0 & \text{on} & \partial\Omega. \end{array}\right\}$$

where $\Delta_{R,r}$ with $r > 1$ and $R = R(x)$ is a weight function, denotes the weighted r-Laplacian defined by $\Delta_{R,r}u \equiv div[R(x)|\nabla u|^{r-2}\nabla u]$, λ is a positive parameter, $a(x)$, $b(x)$, $c(x)$ and $d(x)$ are weight functions, $\alpha, \beta \geqslant 0$, γ, $\delta > 0$, $\alpha + \delta < p - 1$, $\beta + \gamma < q - 1$ and $\Omega \subset \Re^N$ is a bounded domain with smooth boundary $\partial\Omega$. We use the method of sub-supersolutions to establish our results.

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1. Introduction

Many authors are interested in the study of the existence, nonexistence and stability of positive weak solutions of nonlinear systems involving weighted p-Laplacian during recent years (see [2, 4, 5, 12, 13, 20] and the references therein).

Khafagy in [15] proved the existence, nonexistence and uniqueness of positive weak solution for the nonlinear system

$$\left\{\begin{array}{rll} -\Delta_{P,p}u = \lambda a(x)u^{\alpha} & \text{in} & \Omega \\ u > k & \text{in} & \Omega \\ u = k & \text{on} & \partial\Omega, \end{array}\right. \tag{1.1}$$

where $0 < \alpha < p - 1$, using the method of sub-supersolutions. These results extended in [8] to the nonlinear system

$$\left\{\begin{array}{rll} -\Delta_{P,p}u = a(x)(\lambda u^{\alpha} + u^{\beta}) & \text{in} & \Omega \\ u > k & \text{in} & \Omega \\ u = k & \text{on} & \partial\Omega, \end{array}\right. \tag{1.2}$$

wehere $0 < \beta \leq \alpha < p - 1$.

In [7] the author studied the existence and nonexistence of positive weak solutions for the weighted $(p, q)-$Laplacian nonlinear system

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$$\left.\begin{aligned} -\Delta_{P,p} u &= \lambda a(x) v^{\beta} && \text{in } \Omega, \\ -\Delta_{Q,q} v &= \lambda b(x) u^{\alpha} && \text{in } \Omega, \\ u &= v = 0 && \text{on } \partial\Omega, \end{aligned}\right\} \tag{1.3}$$

where $0 < \alpha < p - 1$, $0 < \beta < q - 1$, using the the sub–supersolutions method.

In this paper, we are concerned with the existence and nonexistence of positive weak solutions for the nonlinear system

$$\left.\begin{aligned} -\Delta_{P,p} u &= \lambda a(x) v^{\gamma} + b(x) u^{\alpha} && \text{in } \Omega, \\ -\Delta_{Q,q} v &= \lambda c(x) u^{\delta} + d(x) v^{\beta} && \text{in } \Omega, \\ u &= v = 0 && \text{on } \partial\Omega, \end{aligned}\right\} \tag{1.4}$$

where $\Delta_{R,r}$ with $r > 1$ and $R = R(x)$ is a weight function, $R(x) = P(x)$ when $r = p$ and $R(x) = Q(x)$ when $r = q$, denotes the weighted r-Laplacian defined by $\Delta_{R,r} u \equiv div[R(x)|\nabla u|^{r-2}\nabla u]$, λ is a positive parameter, $a(x)$, $b(x)$, $c(x)$ and $d(x)$ are weight functions, $\alpha, \beta \geqslant 0$, γ, $\delta > 0$, $\alpha + \delta < p - 1$, $\beta + \gamma < q - 1$ and $\Omega \subset \Re^N$ is a bounded domain with smooth boundary $\partial\Omega$. We discuss, under some certian conditions, the existence of positive weak solution for system (1.4). We also study the conditions under which system (1.4) has no positive weak solution. The existence results are obtained by the sub-supersolutions method.

Clearly, our work extends the work in (1.1)-(1.3) by considering a problem with a more general class of functions. Therefore we have a generalization of all results obtained for the above systems.

On the other hand, the existence of weak solutions for nonlinear systems involving p-Laplacian operators with different weights have been studied using, the sub-supersolutions method (see [8, 10, 11, 14, 15]) the theory of nonlinear monotone operators method (see [16, 21]) and the approximation method (see [22, 23]).

Finally, let us explain the plan of the paper. In section 2, we introduce some technical results and notations, which are established in [6]. In section 3, we prove the existence of a positive weak solutions for system (1.4) by using the method of sub–supersolutions. In section 4, we consider the nonexistence results.

2. Technical Results

In this section, we introduce some technical results [6] concerning the degenerated homogeneous eigenvalue problem

$$\left.\begin{aligned} -\Delta_{R,r} u = -div[R(x)|\nabla u|^{r-2}\nabla u] &= \lambda S(x)|u|^{r-2}u && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}\right\} \tag{2.1}$$

where $R(x)$ is a weight function (measurable and positive a.e. in Ω), satisfying the conditions

$$R(x) \in L^1_{Loc}(\Omega), \; (R(x))^{-\frac{1}{r-1}} \in L^1_{Loc}(\Omega), \text{ with } r > 1, \tag{2.2}$$

$$(R(x))^{-s} \in L^1(\Omega), \quad \text{with } s \in \left(\frac{N}{r}, \infty\right) \cap \left[\frac{1}{r-1}, \infty\right), \tag{2.3}$$

and $S(x)$ is a measurable function satisfies

$$S(x) \in L^{\frac{k}{k-r}}(\Omega), \tag{2.4}$$

with some k satisfies $r < k < r_s^*$ where $r_s^* = \frac{Nr_s}{N-r_s}$ with $r_s = \frac{rs}{s+1} < r < r_s^*$ and $meas\ \{x \in \Omega : S(x) > 0\} > 0$.

Lemma 2.1. [6] *There exists the least(i.e. the first or principal) eigenvalue $\lambda_1^{(r)} > 0$ and precisely one corresponding eigenfunction $\phi_{1,r} \geq 0$ a.e. in Ω ($\phi_{1,r}$ not identical to 0) of the eigenvalue problem (2.1).*

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Moreover, it is characterized by

$$\lambda_1^{(r)} \int_\Omega S(x)\phi_{1,r}^r \leq \int_\Omega R(x)|\nabla \phi_{1,r}^r|^p. \tag{2.5}$$

Lemma 2.2. [6] *Let $\phi_{1,r} \geq 0$ a.e. in Ω, be the eigenfunction corresponding to the first eigenvalue $\lambda_1^{(r)} > 0$ of the eigenvalue problem (2.1). Then $\phi_{1,r} \in L^\infty(\Omega)$.*

Now, let us introduce the weighted Sobolev space $W^{1,p}(R,\Omega)$ which is the set of all real valued functions u defined in Ω for which (see [6])

$$\|u\|_{W^{1,r}(R,\Omega)} = \left[\int_\Omega |u|^r + \int_\Omega R(x)|\nabla u|^r\right]^{\frac{1}{r}} < \infty. \tag{2.6}$$

Since we are dealing with the Dirichlet problem, we introduce also the space $W_0^{1,r}(R,\Omega)$ as the closure of $C_0^\infty(\Omega)$ in $W^{1,r}(R,\Omega)$ with respect to the norm

$$\|u\|_{W_0^{1,r}(R,\Omega)} = \left[\int_\Omega R(x)|\nabla u|^r\right]^{\frac{1}{r}} < \infty, \tag{2.7}$$

which is equivalent to the norm given by (2.6). Both spaces $W^{1,r}(R,\Omega)$ and $W_0^{1,r}(R,\Omega)$ are well defined reflexive Banach Spaces.

3. Existence Results

We shall prove the existence of a positive weak solution for system (1.4) by constructing a subsolution $(\psi_1,\psi_2) \in W_0^{1,p}(P,\Omega) \times W_0^{1,q}(Q,\Omega)$ and a supersolution $(z_1,z_2) \in W_0^{1,p}(P,\Omega) \times W_0^{1,q}(Q,\Omega)$ of (1.4) such that $\psi_i \leq z_i$ for $i = 1,2$. That is, ψ_i, $i = 1,2$, satisfies

$$\begin{aligned}
\int_\Omega P(x)|\nabla \psi_1|^{p-2}\nabla\psi_1\nabla\zeta dx &\leq \lambda\int_\Omega a(x)\psi_2^\gamma\zeta dx + \int_\Omega b(x)\psi_1^\alpha\zeta dx,\\
\int_\Omega Q(x)|\nabla \psi_2|^{q-2}\nabla\psi_2\nabla\eta dx &\leq \lambda\int_\Omega c(x)\psi_1^\delta\eta dx + \int_\Omega d(x)\psi_2^\beta\eta dx,
\end{aligned}$$

and z_i, $i = 1,2$, satisfies

$$\begin{aligned}
\int_\Omega P(x)|\nabla z_1|^{p-2}\nabla z_1\nabla\zeta dx &\geq \lambda\int_\Omega a(x)z_2^\gamma\zeta dx + \int_\Omega b(x)z_1^\alpha\zeta dx,\\
\int_\Omega Q(x)|\nabla z_2|^{q-2}\nabla z_2\nabla\eta dx &\geq \lambda\int_\Omega c(x)z_1^\delta\eta dx + \int_\Omega d(x)z_2^\beta\eta dx,
\end{aligned}$$

for all test functions $\zeta \in W_0^{1,p}(P,\Omega)$ and $\eta \in W_0^{1,q}(Q,\Omega)$ with $\zeta,\eta \geq 0$.

Lemma 3.1. *(see* [3, 7]*) Suppose there exist sub and supersolutions (ψ_1,ψ_2) and (z_1,z_2) respectively of (1.4) such that $(\psi_1,\psi_2) \leq (z_1,z_2)$. Then (1.4) has a solution (u,v) such that $(u,v) \in [(\psi_1,\psi_2),(z_1,z_2)]$.*

Theorem 3.2. *Let $(p-1)(q-1) - \gamma\delta > 0$. Then system (1.4) has a positive weak solution (u,v) for each $\lambda > 0$.*

Proof. Let $\lambda_1^{(r)}$ be the first eigenvalue of the eigenvalue problem (2.1) and $\phi_{1,r}$ the corresponding positive eigenfunction satisfying $\phi_{1,r} > 0$ in Ω and $|\nabla \phi_{1,r}| > 0$ on $\partial\Omega$ with $\|\phi_{1,r}\|_\infty = 1$, for $r = p, q$. Then we have

$$\left.\begin{array}{llll} -\Delta_{P,p}\phi_{1,p} = \lambda_1^{(p)} a(x)|\phi_{1,p}|^{p-2}\phi_{1,p} & \text{in}\,\Omega, & \phi_{1,p} = 0 & \text{on }\partial\Omega, \\ -\Delta_{Q,q}\phi_{1,q} = \lambda_1^{(q)} c(x)|\phi_{1,q}|^{q-2}\phi_{1,q} & \text{in}\,\Omega, & \phi_{1,q} = 0 & \text{on }\partial\Omega. \end{array}\right\} \tag{3.1}$$

Since $(p-1)(q-1) - \gamma\delta > 0$, we can take k such that

$$\frac{\delta}{q-1} < k < \frac{p-1}{\gamma}. \tag{3.2}$$

We shall verify that $(\psi_1, \psi_2) = (\ \xi\phi_{1,p}^{\frac{p}{p-1}}, \xi^k \phi_{1,q}^{\frac{q}{q-1}})$ is a subsolution of (1.4), where $\xi > 0$ is small and specified later. Let $\zeta \in W_0^{1,p}(P,\Omega)$ with $\zeta \geq 0$. A calculation shows that

$$\begin{aligned} \int_\Omega P(x)|\nabla\psi_1|^{p-2}\nabla\psi_1 \cdot \nabla\zeta dx &= (\frac{p}{p-1}\xi)^{p-1}\int_\Omega P(x)\phi_{1,p}|\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\cdot\nabla\zeta dx \\ &= (\frac{p}{p-1}\xi)^{p-1}\int_\Omega (P(x)|\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\nabla(\phi_{1,p}\zeta) - P(x)|\nabla\phi_{1,p}|^p)\zeta dx \\ &= (\frac{p}{p-1}\xi)^{p-1}\int_\Omega (\lambda_1^{(p)}a(x)\phi_{1,p}^p - P(x)|\nabla\phi_{1,p}|^p)\zeta dx. \end{aligned}$$

Similarly, for $\eta \in W_0^{1,q}(Q,\Omega)$ with $\eta \geq 0$, we have

$$\int_\Omega Q(x)|\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla\eta dx = (\frac{q}{q-1}\xi^k)^{q-1}\int_\Omega (\lambda_1^{(q)}c(x)\phi_{1,q}^q - Q(x)|\nabla\phi_{1,q}|^q)\eta dx.$$

Since $\phi_{1,r} = 0$ and $|\nabla\phi_{1,r}| > 0$ on $\partial\Omega$, there is $\epsilon > 0$ such that

$$\lambda_1^{(p)}a(x)\phi_{1,p}^p - P(x)|\nabla\phi_{1,p}|^p \ \leq 0 \quad \text{and} \quad \lambda_1^{(q)}c(x)\phi_{1,q}^q - Q(x)|\nabla\phi_{1,q}|^q \ \leq 0 \qquad \text{on } \overline{\Omega}_\epsilon$$

with $\overline{\Omega}_\epsilon = \{x \in \Omega : d(x, \partial\Omega) \leq \epsilon\}$. This shows that

$$(\frac{p}{p-1}\xi)^{p-1}\int_{\overline{\Omega}_\epsilon}(\lambda_1^{(p)}a(x)\phi_{1,p}^p - P(x)|\nabla\phi_{1,p}|^p)\zeta dx \leq 0 \leq \lambda\int_{\overline{\Omega}_\epsilon} a(x)\psi_2^\gamma\zeta dx + \int_{\overline{\Omega}_\epsilon} b(x)\psi_1^\alpha\zeta dx,$$

and

$$(\frac{q}{q-1}\xi^k)^{q-1}\int_{\overline{\Omega}_\epsilon}(\lambda_1^{(q)}c(x)\phi_{1,q}^q - Q(x)|\nabla\phi_{1,q}|^q)\eta dx \leq 0 \leq \lambda\int_{\overline{\Omega}_\epsilon} c(x)\psi_1^\delta\eta dx + \int_{\overline{\Omega}_\epsilon} d(x)\psi_2^\beta\eta dx.$$

Furthermore, we note that $\phi_{1,p}$, $\phi_{1,q} \geq \sigma$ in $\Omega - \overline{\Omega}_\epsilon$ for some $\sigma > 0$. Then from (3.2) there is $\xi_0 > 0$ such that if $\xi \in (0, \xi_0)$, the following inequalities hold:

$$\xi^{p-1-k\gamma}(\frac{p}{p-1})^{p-1}\lambda_1^{(p)}a(x)\phi_{1,p}^p \leq \lambda a(x)\ \sigma^{\frac{\gamma q}{q-1}} \leq \lambda a(x)\ \phi_{1,q}^{\frac{\gamma q}{q-1}} \qquad \text{in } \Omega - \overline{\Omega}_\epsilon,$$

and

$$\xi^{k(q-1)-\delta}\ (\frac{q}{q-1})^{q-1}\ \lambda_1^{(q)}c(x)\phi_{1,q}^q\ \leq \lambda c(x)\sigma^{\frac{\delta p}{p-1}} \leq \lambda c(x)\phi_{1,p}^{\frac{\alpha p}{p-1}} \quad \text{in } \Omega - \overline{\Omega}_\epsilon.$$

Then, we have

$$\begin{aligned}\int\limits_{\Omega-\overline{\Omega}_\epsilon} P(x)|\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla\zeta dx &= (\frac{p}{p-1}\xi)^{p-1}\int\limits_{\Omega-\overline{\Omega}_\epsilon}(\lambda_1^{(p)}a(x)\phi_{1,p}^p - P(x)|\nabla\phi_{1,p}|^p)\zeta dx\\ &\leq \lambda\int\limits_{\Omega-\overline{\Omega}_\epsilon} a(x)\xi^{\gamma k}\phi_{1,q}^{\frac{\gamma q}{q-1}}\zeta dx \leq \lambda\int\limits_{\Omega-\overline{\Omega}_\epsilon} a(x)\psi_2^\gamma\zeta dx + \int\limits_{\Omega-\overline{\Omega}_\epsilon} b(x)\psi_1^\alpha\zeta dx.\end{aligned}$$

Similarly,

$$\begin{aligned}\int\limits_{\Omega-\overline{\Omega}_\epsilon} Q(x)|\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla\eta dx &= (\frac{q}{q-1}\xi^k)^{q-1}\int\limits_{\Omega-\overline{\Omega}_\epsilon}(\lambda_1^{(q)}c(x)\phi_{1,q}^q - Q(x)|\nabla\phi_{1,q}|^q)\eta dx\\ &\leq \lambda\int\limits_{\Omega-\overline{\Omega}\epsilon} c(x)\xi^{\delta}\phi_{1,p}^{\frac{\delta p}{p-1}}\eta dx \leq \lambda\int\limits_{\Omega-\overline{\Omega}\epsilon} c(x)\psi_1^\delta\eta dx + \int\limits_{\Omega-\overline{\Omega}\epsilon} d(x)\psi_2^\alpha\eta dx,\end{aligned}$$

i.e. (ψ_1,ψ_2) is a subsolution of (1.4).

Next, we construct a supersolution (z_1,z_2) of system (1.4). Let $e_r(x)$ be the positive solution of (see [23])

$$-\Delta_{R,r}e_r = 1 \quad \text{in } \Omega, \qquad e_r = 0 \qquad \text{on } \partial\Omega \qquad \text{for } r = p,q.$$

We denote $z_1(x) = Ae_p,\quad z_2(x) = Be_q,$ where the constants $A,\ B > 0$ are large and to be chosen later. We shall verify that (z_1,z_2) is the supersolution of (1.4). To do this, let $\zeta\in W_0^{1,p}(P,\Omega)$ with $\zeta\geq 0$. Then we have

$$\int\limits_\Omega P(x)|\nabla z_1|^{p-2}\nabla z_1\cdot\nabla\zeta dx = A^{p-1}\int\limits_\Omega P(x)|\nabla e_p|^{p-2}\nabla e_p\cdot\nabla\zeta dx = A^{p-1}\int\limits_\Omega \zeta dx.$$

Similarly, for $\eta\in W_0^{1,q}(Q,\Omega)$ with $\eta\geq 0$, we have

$$\int\limits_\Omega Q(x)|\nabla z_2|^{q-2}\nabla z_2\cdot\nabla\eta dx = B^{q-1}\int\limits_\Omega Q(x)|\nabla e_q|^{q-2}\nabla e_q\cdot\nabla\eta dx = B^{q-1}\int\limits_\Omega \eta dx.$$

Since $(p-1)(q-1)-\gamma\delta > 0$, it is easy to prove that there exist positive large constants A, B such that

$$A^{p-1} = \lambda l_a B^\gamma\mu_q^\gamma + l_b A^\alpha\mu_p^\alpha, \qquad B^{q-1} = \lambda l_c A^\delta\mu_p^\delta + l_d B^\beta\mu_q^\beta,$$

where $l_a = |a(x)|,\ l_b = |b(x)|,\ l_c = |c(x)|,\ l_d = |d(x)|$ and $\mu_r = \|e_r\|_\infty$; $r = p,q$. These imply that

$$\begin{aligned}\int\limits_\Omega P(x)|\nabla z_1|^{p-2}\nabla z_1\cdot\nabla\zeta dx &= \lambda\int\limits_\Omega l_a B^\gamma\mu_q^\gamma\zeta dx + \int\limits_\Omega l_b A^\alpha\mu_p^\alpha\zeta dx\\ &\geq \lambda\int\limits_\Omega a(x)z_2^\gamma\zeta dx + \int\limits_\Omega b(x)z_1^\alpha\zeta dx,\end{aligned}$$

and

$$\begin{aligned}\int\limits_\Omega Q(x)|\nabla z_2|^{q-2}\nabla z_2\cdot\nabla\eta dx &= \lambda\int\limits_\Omega l_c A^\delta\mu_p^\delta\eta dx + \int\limits_\Omega l_d B^\beta\mu_q^\beta\eta dx\\ &\geq \lambda\int\limits_\Omega c(x)z_1^\delta\eta dx + \int\limits_\Omega d(x)z_2^\beta\eta dx,\end{aligned}$$

i.e. (z_1,z_2) is a supersolution of (1.4) with $z_i\geq\psi_i$ with large A,B, for $i=1,2$. Thus, there exists a solution (u,v) of (1.4) with $\psi_1\leq u\leq z_1,\ \ \psi_2\leq v\leq z_2$. This completes the proof.

□

4. Nonexistence result

In this section, under some certian conditions we prove that system (1.4) has no positive weak solution.

Theorem 4.1. *Suppose that* $(p-1)(q-1)-\gamma\delta = 0$, $p\gamma = q(p-1)$ *and* $b(x)$, $d(x) \le a(x) = c(x)$ *for all* $x \in \Omega$. *Then there exists* $\lambda_0 > 0$ *such that for* $0 < \lambda < \lambda_0$, *system (1.4) has no positive weak solution.*

Proof. Let us assume that $(u, v) \in W_0^{1,p}(P, \Omega) \times W_0^{1,p}(Q, \Omega)$ be a positive weak solution of (1.4). We prove this theorem by arriving at a contradiction.

Multiplying the first equation of (1.4) by u, we have from Young inequality that

$$\int_\Omega P(x)|\nabla u|^p dx \le \lambda \int_\Omega a(x)(\frac{u^p}{\mu_1} + \frac{v^q}{\mu_2})dx + \int_\Omega b(x)u^p, \tag{4.1}$$

with $\mu_1 = p > 1$ and $\mu_2 = \frac{p}{p-1} > 1$.

Similarly, we have

$$\int_\Omega Q(x)|\nabla v|^q dx \le \lambda \int_\Omega c(x)(\frac{u^p}{\theta_1} + \frac{v^q}{\theta_2})dx + \int_\Omega d(x)v^q, \tag{4.2}$$

with $\theta_1 = \frac{q}{q-1} > 1$ and $\theta_2 = q > 1$.

Note that

$$\lambda_1^{(p)} \int_\Omega a(x)u^p dx \le \int_\Omega P(x)|\nabla u|^p dx, \quad \lambda_1^{(q)} \int_\Omega c(x)v^q dx \le \int_\Omega Q(x)|\nabla v|^q dx. \tag{4.3}$$

Combining (4.1)-(4.3), we obtain

$$\lambda_1^{(p)} \int_\Omega a(x)u^p dx + \lambda_1^{(q)} \int_\Omega c(x)v^q dx \le \lambda \int_\Omega (\frac{a(x)}{\mu_1} + \frac{c(x)}{\theta_1})u^p dx + \lambda \int_\Omega (\frac{a(x)}{\mu_2} + \frac{c(x)}{\theta_2})v^q dx + \int_\Omega b(x)u^p + \int_\Omega d(x)v^q.$$

Now, if $b(x)$, $d(x) \le a(x) = c(x)$ for all $x \in \Omega$, we have

$$(\lambda_1^{(p)} - 1 - \lambda) \int_\Omega a(x)u^p dx + (\lambda_1^{(q)} - 1 - \lambda) \int_\Omega a(x)v^q dx \le 0,$$

which is a contradiction if $0 < \lambda < \lambda_0 = \min\{\lambda_1^{(p)} - 1, \lambda_1^{(q)} - 1\}$. This completes the proof. $\square$

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