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TRAVELING WAVE SOLUTIONS OF SOME FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. The modified Kudryashov method is powerful, efficient and can be used as an alternative to establish new solutions of different type of fractional differential equations applied in mathematical physics. In this article, we've constructed new traveling wave solutions including symmetrical Fibonacci function solutions, hyperbolic function solutions and rational solutions of the space-time fractional Cahn Hillihard equation

$$D_t^\alpha u - \gamma D_x^\alpha u - 6u(D_x^\alpha u)^2 - (3u^2-1)D_x^\alpha(D_x^\alpha u) + D_x^\alpha(D_x^\alpha(D_x^\alpha(D_x^\alpha u))) = 0$$

and the space-time fractional symmetric regularized long wave (SRLW) equation

$$D_t^\alpha(D_t^\alpha u) + D_x^\alpha(D_x^\alpha u) + uD_t^\alpha(D_x^\alpha u) + D_x^\alpha u D_t^\alpha u + D_t^\alpha(D_t^\alpha(D_x^\alpha(D_x^\alpha u))) = 0$$

via modified Kudryashov method. In addition, some of the solutions are described in the figures with the help of Mathematica.

Mathematics Subject Classification (2010): 35Q79; 35Q35; 35Q40.
Key words: Modified Kudryashov method, fractional partial differential equations, the space-time fractional Cahn Hillihard equation, the space-time fractional symmetric regularized long wave (SRLW) equation.

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1. Introduction

In the past, the theory of fractional derivative, only in the interest of mathematicians, was seen as the subject of pure mathematics. However, in the last few decades, fractional phenomena attracted the attention of scientists in many other fields. Many researchers indicated that integrals and derivatives with non-integer order are very beneficial to describe the structure of various real materials. There exist many fractional derivatives in literature but few of them commonly used, including Caputo, Riemann-Liouville and modified Riemann-Liouville [28],[30]. Caputo derivatives are defined only for differentiable functions, while the functions can be continuous (but not necessarily differentiable) in the Riemann-Liouville sense. Riemann-Liouville definition can be used for any functions that are continuous but not differentiable anywhere, however, the derivative of a constant is not zero [11]. To overcome the shortcomings, the modification of Riemann-Liouville fractional derivative for continuous (but not necessarily differentiable) functions [11],[18],[19] is suggested. Fractional derivatives and integrals become apparent in dynamical systems which are described by fractional differential equations (FDEs)[28],[30]. FDEs also appears in physics, chemistry, mathematical biology, electromagnetic theory, fluid mechanics, quantum mechanics, engineering and other fields of science. Many effective methods have been proposed to find the numerical and analytical solutions of fractional partial differential equations. Such as, the differential transform method [29], the Adomian's decomposition method [3],[8],[14], the variational iteration method [7],[15],[34], the sub-equation method [10],[17],[26],[27],[36],[37], the extended fractional Riccati expansion

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method [24], the exp-function method [9],[38], the first integral method [25],[35], the iterative Laplace transform method [16], (G'/G) -expansion method [1],[2],[9],[31], extended Jacobi elliptic function expansion method [32] and others. In 2012, R.N. Kudryashov [23] proposed a method to obtain the analytical solutions of nonlinear partial differential equations and the useful modified Kudryashov method have been used by many authors [4]-[6],[21],[22]. This method is based on the modified Riemann Liouville sense derivative and the homogenous balance principle. In this method, by using the transformation $\xi = \xi(x, y, z, \ldots, t)$, a given fractional order differential equation turn into fractional ordinary differential equation whose solutions are in the form $u(\xi) = \sum_{i=0}^{M} a_i Y^i(\xi)$, where $Y(\xi)$ satisfies the fractional Riccati equation $Y_\xi = \ln a(Y^2 - Y)$. In previous studies, many authors used different nonlinear fractional traveling wave transformation was taken for ξ, then a certain fractional equation turned into another integer order ordinary differential equation, which allowed the use of the integer order Riccati equation instead of the fractional Riccati equation in [12],[13]. Therefore, we notice that the modified Kudryashov method is suitable for solving partial differential equation of fractional order involving fractional partial derivatives of certain orders. In this study, we will apply the modified Kudryashov method for solving some fractional order partial differential equations. To illustrate the validity of this method we will apply it to the space-time fractional space-time fractional SRLW equation and the space-time fractional Cahn Hillihard equation.

2. Preliminaries

Definition 2.1. A real function $f(t)$, $t > 0$, is said to be in the space C_κ, $\kappa \in R$, if there exists a real number $p > \kappa$ such that $f(t) = t^p f_1(t)$, where $f_1(t) \in C(0, \infty)$, and it is said to be in the space C_κ^m if $f^m \in C_\kappa, m \in N$ [18],[19].

Definition 2.2. The modified Riemann-Liouville derivative is defined as [18]-[20]:

$$D_x^\alpha f(x) := \lim_{h \downarrow 0} h^{-\alpha} \sum_{k=0}^{\infty} (-1)^k \binom{\alpha}{k} f[x + (\alpha - k)h]. \tag{2.1}$$

If f is not a constant, then it follows that

$$D_x^\alpha f(x) = \begin{cases} \dfrac{1}{\Gamma(-\alpha)} \displaystyle\int_0^x (x - \xi)^{-\alpha - 1} f(\xi) d\xi, \text{ for } \alpha < 0, \\ \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dx} \displaystyle\int_0^x (x - \xi)^{-\alpha} [f(\xi) - f(0)] \, d\xi, \text{ for } 0 < \alpha < 1, \\ \left(D_x^{\alpha - n}(x)\right)^{(n)}, \text{ for } n \le \alpha < n + 1, \quad n \ge 1. \end{cases} \tag{2.2}$$

Moreover, some properties for the proposed modified Riemann-Liouville derivative are given in [11] as follows:

$$D_t^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} t^{\gamma - \alpha}, \quad \gamma > 0, \tag{2.3}$$

$$D_t^\alpha c = 0, \tag{2.4}$$

$$D_t^\alpha (c_1 f(t) + c_2 g(t)) = c_1 D_t^\alpha f(t) + c_2 D_t^\alpha g(t), \tag{2.5}$$

where c, c_1, c_2 are constant.

We present the main steps of the modified Kudryashov method (see [4]-[6],[21]-[23]) as follows: Consider a nonlinear FDE for a function u of independent variables, $x, y, z, \ldots, t$:

$$P\left(u, D_t^\alpha u, D_x^\beta u, D_y^\gamma u, D_z^\delta u, D_t^\alpha (D_t^\alpha u), D_t^\alpha (D_x^\alpha u), D_t^\alpha (D_z^\alpha u), \ldots\right) = 0, \tag{2.6}$$

where $D_t^\alpha u, D_x^\beta u, D_y^\gamma u$ and $D_z^\delta u$ are the modified Riemann-Liouville derivatives of u with respect to t, x, y and z. P is a polynomial in $u = u(x, y, z, \ldots, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved.

Step 1. We investigate the traveling wave solutions of (2.6) by making the transformations presented by [11]-[13] in the form :

$$u(x, y, z, \ldots, t) = u(\xi), \quad \xi = \frac{kx^\beta}{\Gamma(1+\beta)} + \frac{ny^\gamma}{\Gamma(1+\gamma)} + \frac{mz^\delta}{\Gamma(1+\delta)} + \cdots + \frac{\lambda t^\alpha}{\Gamma(1+\alpha)}, \tag{2.7}$$

where k, n, m and λ are arbitrary constants. By using the equation (see [20], Corollary 4.1)

$$D_x^\alpha f(u(x)) = f'_u D_x^\alpha u,$$

and by (2.4), under suitable hypotheses (see [12]), (2.6) becomes

$$G(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0, \tag{2.8}$$

where $u = u(\xi)$.

Step 2. We suppose that the reduced equation admits the following solution:

$$u(\xi) = \sum_{i=0}^{M} a_i Y^i(\xi) \tag{2.9}$$

where $Y = \frac{1}{1 \pm a^\xi}$ and the function Y is the solution of equation

$$Y_\xi = \ln a(Y^2 - Y). \tag{2.10}$$

Step 3. According to the method, we assume that the solution of (2.8) can be expressed in the form (2.9). In order to determine the value of the pole order M, we balance the highest order nonlinear terms in (2.8) analogously as in the classical Kudryashov method. Supposing $u^l(\xi)u^{(s)}(\xi)$ and $(u^{(p)}(\xi))^r$ are the highest order nonlinear terms of (2.8) and balancing the highest order nonlinear terms we have:

$$M = \frac{s - rp}{r - l - 1}. \tag{2.11}$$

Step 4. Substituting (2.9) into (2.8) and equating the coefficients of Y^i to zero, we get a system of algebraic equations. By solving this system, we obtain exact solutions of (2.6), and the obtained solutions can depend on symmetrical hyperbolic Fibonacci functions proposed by Stakhov and Rozin [33]. Symmetrical Fibonacci sine, cosine, tangent, cotangent functions are respectively defined as:

$$\mathrm{sFs}(x) = \frac{a^x - a^{-x}}{\sqrt{5}}, \quad \mathrm{cFs}(x) = \frac{a^x + a^{-x}}{\sqrt{5}}, \tag{2.12}$$

$$\mathrm{tanFs}(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}, \quad \mathrm{cotFs}(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}. \tag{2.13}$$

3. Applications

Example 3.1. We consider the following the space-time fractional Cahn-Hillihard equation:

$$D_t^\alpha u - \gamma D_x^\alpha u - 6u(D_x^\alpha u)^2 - (3u^2 - 1)D_x^\alpha(D_x^\alpha u) + D_x^\alpha(D_x^\alpha(D_x^\alpha(D_x^\alpha u))) = 0 \tag{3.1}$$

where $0 < \alpha \leq 1$. By considering the traveling wave transformation

$$u(x, t) = u(\xi), \xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} + \xi_0 \tag{3.2}$$

where k, l, c, ξ_0 are constants, equation (3.1) can be reduced to the following ordinary differential equation:

$$(c - k\gamma)u' - 6k^2u(u')^2 - 3k^2u^2u'' + k^2u'' + k^4u^{(4)} = 0. \tag{3.3}$$

Also we take

$$u(\xi) = a_0 + a_1 Y + \cdots + a_M Y^M \tag{3.4}$$

where $Y = \frac{1}{1 \pm a^{\xi}}$. We note that the function Y is the solution of $Y_\xi = \text{ln}a(Y^2 - Y)$. Balancing the linear term of the highest order with the highest order nonlinear term in (3.3), we compute

$$M = 1. \tag{3.5}$$

Thus, we have

$$u(\xi) = a_0 + a_1 Y \tag{3.6}$$

and substituting derivatives of $u(\xi)$ with respect to ξ in (3.6) we obtain

$$u_\xi = \text{ln}a(a_1 Y^2 - a_1 Y), \tag{3.7}$$

$$u_{\xi\xi} = (\text{ln}a)^2(2a_1 Y^3 - 3a_1 Y^2 + a_1 Y). \tag{3.8}$$

$$u_{\xi\xi\xi} = (\text{ln}a)^3(6a_1 Y^4 - 12a_1 Y^3 + 7a_1 Y^2 - a_1 Y). \tag{3.9}$$

$$u_{\xi\xi\xi\xi} = (\text{ln}a)^4(24a_1 Y^5 - 60a_1 Y^4 + 50a_1 Y^3 - 15a_1 Y^2 + a_1 Y). \tag{3.10}$$

Substituting (3.7)-(3.10) into (3.3) and collecting the coefficient of each power of Y^i, setting each of coefficient to zero, solving the resulting system of algebraic equations we obtain the following solutions:
Case 1:

$$a_0 = 1, a_1 = -2, k = -\frac{\sqrt{2}}{\text{ln}a}, c = -\frac{\sqrt{2}\gamma}{\text{ln}a}. \tag{3.11}$$

Inserting (3.11) into (3.6), we obtain the following solutions of (3.1)

$$u_1(x,t) = \text{tanFs}\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2(\text{ln}a)\Gamma(1+\alpha)}\right), \tag{3.12}$$

$$u_2(x,t) = \text{cotFs}\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2(\text{ln}a)\Gamma(1+\alpha)}\right). \tag{3.13}$$

For $a = e$

$$u_3(x,t) = \tanh\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2\Gamma(1+\alpha)}\right), \tag{3.14}$$

$$u_4(x,t) = \coth\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2\Gamma(1+\alpha)}\right). \tag{3.15}$$

Case 2:

$$a_0 = -1, a_1 = 2, k = -\frac{\sqrt{2}}{\text{ln}a}, c = -\frac{\sqrt{2}\gamma}{\text{ln}a}. \tag{3.16}$$

Inserting (3.16) into (3.6), we obtain the following solutions of (3.1)

$$u_5(x,t) = -\text{tanFs}\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2(\text{ln}a)\Gamma(1+\alpha)}\right), \tag{3.17}$$

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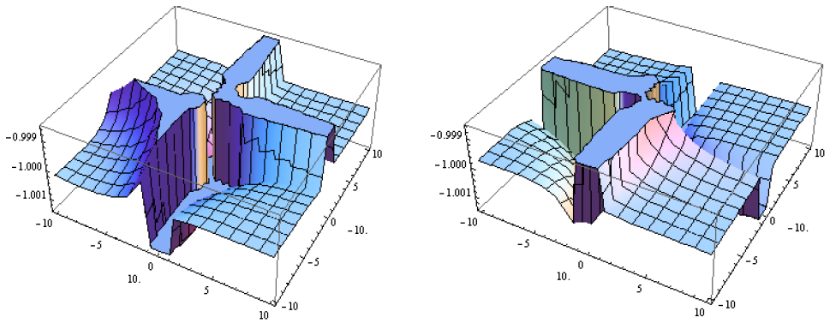

FIGURE 1. Solitary wave solutions of (3.1), $u_1(x,t)$, $u_2(x,t)$ are shown at $a = 10$ and $\alpha = 0.25$.

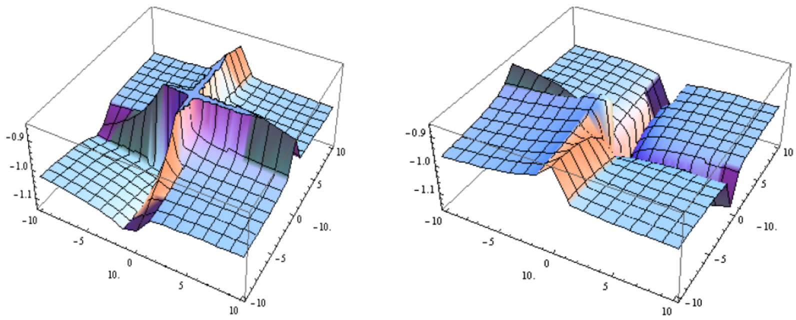

FIGURE 2. Solitary wave solution of (3.1), $u_3(x,t)$, $u_4(x,t)$ are shown at $a = e$ and $\alpha = 0.25$.

$$u_6(x,t) = -\text{cotFs}\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2(\ln a)\Gamma(1+\alpha)}\right). \tag{3.18}$$

For $a = e$

$$u_7(x,t) = -\tanh\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2\Gamma(1+\alpha)}\right), \tag{3.19}$$

$$u_8(x,t) = -\coth\left(\frac{-\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2\Gamma(1+\alpha)}\right). \tag{3.20}$$

Case 3:

$$a_0 = 1, a_1 = -2, k = -\frac{\sqrt{2}}{\ln a}, c = \frac{\sqrt{2}\gamma}{\ln a}. \tag{3.21}$$

Inserting (3.21) into (3.6), we obtain the following solutions of (3.1)

$$u_9(x,t) = \text{tanFs}\left(\frac{\sqrt{2}(x^\alpha + \gamma t^\alpha)}{2(\ln a)\Gamma(1+\alpha)}\right), \tag{3.22}$$

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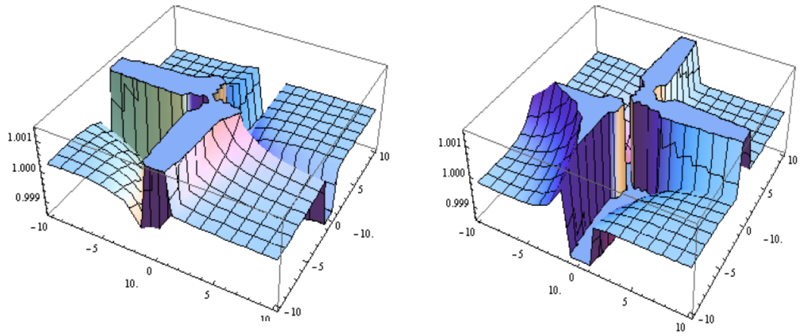

FIGURE 3. Solitary wave solution of (3.1), $u_5(x,t)$, $u_6(x,t)$ are shown at $a = 10$ and $\alpha = 0.25$.

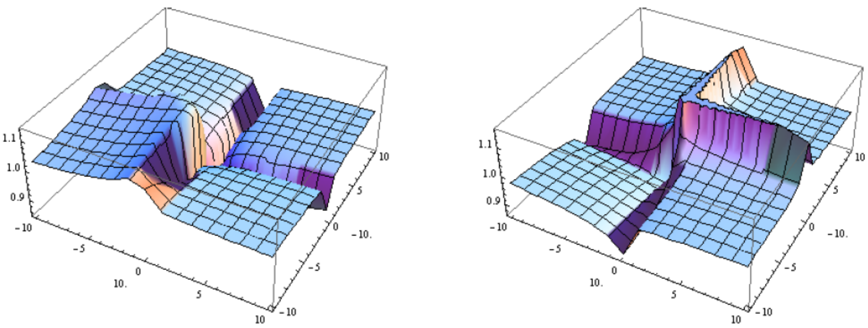

FIGURE 4. Solitary wave solution of (3.1), $u_7(x,t)$, $u_8(x,t)$ are shown at $a = e$ and $\alpha = 0.25$.

$$u_{10}(x,t) = \cot\mathrm{Fs}\left(\frac{\sqrt{2}(x^{\alpha}+\gamma t^{\alpha})}{2(\ln a)\Gamma(1+\alpha)}\right). \tag{3.23}$$

For $a = e$

$$u_{11}(x,t) = \tanh\left(\frac{\sqrt{2}(x^{\alpha}+\gamma t^{\alpha})}{2\Gamma(1+\alpha)}\right), \tag{3.24}$$

$$u_{12}(x,t) = \coth\left(\frac{\sqrt{2}(x^{\alpha}+\gamma t^{\alpha})}{2\Gamma(1+\alpha)}\right). \tag{3.25}$$

Case 4:

$$a_0 = -1, a_1 = 2, k = \frac{\sqrt{2}}{\ln a}, c = \frac{\sqrt{2}\gamma}{\ln a}. \tag{3.26}$$

Inserting (3.26) into (3.6), we obtain the following solutions of (3.1)

$$u_{13}(x,t) = -\tan\mathrm{Fs}\left(\frac{\sqrt{2}(x^{\alpha}+\gamma t^{\alpha})}{2(\ln a)\Gamma(1+\alpha)}\right), \tag{3.27}$$

$$u_{14}(x,t) = -\mathrm{cotFs}\left(\frac{\sqrt{2}(x^\alpha+\gamma t^\alpha)}{2(\ln a)\Gamma(1+\alpha)}\right). \tag{3.28}$$

For $a = e$

$$u_{15}(x,t) = -\tanh\left(\frac{\sqrt{2}(x^\alpha+\gamma t^\alpha)}{2\Gamma(1+\alpha)}\right), \tag{3.29}$$

$$u_{16}(x,t) = -\coth\left(\frac{\sqrt{2}(x^\alpha+\gamma t^\alpha)}{2\Gamma(1+\alpha)}\right). \tag{3.30}$$

Remark 3.2. We obtain analytical results for fractional Cahn Hillihard equation by using modified Riemann-Liouville derivative while Dahmani and Benbachir [3] obtain numerical results by means of Caputo derivative. Our results are different. Hyperbolic function solutions are different from (13) in [17] whereas the solutions become similar in [1] if the coefficients are selected appropriately. In addition, our symmetrical Fibonacci function solutions (3.12), (3.13), (3.17), (3.18), (3.22), (3.23), (3.27), (3.28) are new for the literature.

Example 3.3. We next apply the method to the space-time fractional symmetric regularized long wave (SRLW) equation in the form:

$$D_t^\alpha(D_t^\alpha u) + D_x^\alpha(D_x^\alpha u) + uD_t^\alpha(D_x^\alpha u) + D_x^\alpha u D_t^\alpha u + D_t^\alpha(D_t^\alpha(D_x^\alpha(D_x^\alpha u))) = 0 \tag{3.31}$$

where $0 < \alpha \leq 1$, u is the function of (x,t) and v is a nonzero positive constant. By considering the traveling wave transformation:

$$u(x,t) = u(\xi), \xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} + \xi_0 \tag{3.32}$$

where k, c, ξ_0 are constants and $c \neq 0$. Equation (3.31) can be reduced to the following ordinary differential equation:

$$(c^2+k^2)u'' + kcuu'' + kc(u')^2 + k^2c^2u^{(4)} = 0. \tag{3.33}$$

Also we take

$$u(\xi) = a_0 + a_1 Y + \cdots + a_M Y^M \tag{3.34}$$

where $Y = \frac{1}{1\pm a^\xi}$. We note that the function Y is the solution of $Y_\xi = \ln a(Y^2 - Y)$. Balancing the linear term of the highest order with the highest order nonlinear term in (3.33), we compute

$$M = 2. \tag{3.35}$$

Similarly as in Example 3.3 we get

$$a_0 = -\left(\frac{c}{k} + \frac{k}{c} + ck\right)\ln a, a_1 = -12ck\ln a, a_2 = 12ck\ln a. \tag{3.36}$$

Then we obtain the following solutions of (3.31)

$$u_1(x,t) = \ln a\left(-\left(\frac{c}{k}+\frac{k}{c}+ck\right) + \frac{12ck}{5\mathrm{cFs}^2\left(\frac{kx^\alpha}{2\Gamma(1+\alpha)} + \frac{ct^\alpha}{2\Gamma(1+\alpha)} + \frac{\xi_0}{2}\right)}\right), \tag{3.37}$$

$$u_2(x,t) = \ln a\left(-\left(\frac{c}{k}+\frac{k}{c}+ck\right) - \frac{12ck}{5\mathrm{sFs}^2\left(\frac{kx^\alpha}{2\Gamma(1+\alpha)} + \frac{ct^\alpha}{2\Gamma(1+\alpha)} + \frac{\xi_0}{2}\right)}\right). \tag{3.38}$$

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For $a = e$

$$u_3(x,t) = -\left(\frac{c}{k} + \frac{k}{c} + ck\right) + \frac{3ck}{\cosh^2\left(\frac{kx^\alpha}{2\Gamma(1+\alpha)} + \frac{ct^\alpha}{2\Gamma(1+\alpha)} + \frac{\xi_0}{2}\right)}, \tag{3.39}$$

$$u_4(x,t) = -\left(\frac{c}{k} + \frac{k}{c} + ck\right) - \frac{3ck}{\sinh^2\left(\frac{kx^\alpha}{2\Gamma(1+\alpha)} + \frac{ct^\alpha}{2\Gamma(1+\alpha)} + \frac{\xi_0}{2}\right)}. \tag{3.40}$$

Remark 3.4. Our solution (3.39) is similar to (38) in [9] if the coefficients are selected appropriately. Hyperbolic functions solutions are different from the solutions in [31] and [32]. Furthermore, our symmetrical Fibonacci function solutions (3.37) and (3.38) are new for the literature.

4. Conclusion

In this study, we have obtained new exact analytical solutions including the symmetrical Fibonacci function solutions of space-time fractional Cahn-Hillard equation and the symmetric regularized long wave (SRLW) equation by using the modified Kudryashov method. In special case for $a = e$, we have seen that the symmetrical Fibonacci function solutions turned into hyperbolic function solutions. The obtained hyperbolic solutions of (3.1) and (3.31) are familiar with the solutions obtained in some other researches. As it can be seen that the modified Kudryashov method is based on the homogenous balance principle. Therefore, the method is convenient for solving other type of space-time fractional differential equations in which the homogenous balance principle is satisfied.

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