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MOSTAR INDEX (Mo) AND EDGE Mo INDEX FOR SOME CYCLE RELATED GRAPHS

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Abstract. Topological indices are the numerical descriptors of a molecular structure obtained via molecular graph G. Topological indices are used in structure-property relationship, structure-activity relations and nanotechnology. Also, they hold us to predict certain physicochemical properties such as boiling point, enthalpy of vaporization, stability, and so on. In this study, it was considered the Mostar index and was introduced the edge Mostar index. It was computed mostar index (Mo) and edge Mo index for some cycle related graphs which are wheel graph, gear graph, helm graph, flower graph and friendship graph. Finally, it was compared these results.

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1. INTRODUCTION

Graph theory, which is a branch of discrete mathematics started by solving the problem of the bridges of Königsberg by Leonhard Euler in 1736. Graph theory has attracted attention and gained popularity by the publication of the first book on graph theory (1936). Graph theory has been studied in engineering and science such as physics, biology, computer sciences, chemistry, civil engineering, management, and control.

It takes time and money to find the properties of molecules. To predict the properties of the molecules is achieved by chemical graph theory. The chemical graph theory is focused on finding topological indices. Topological indices are a real number of a molecular structure obtained via molecular graph G whose vertices and edges represent the atoms and the bonds, respectively. They hold us to predict certain physicochemical properties such as boiling point, enthalpy of vaporization, stability, and also are used for studying the properties of molecules such as the structure-property relationship, the structure-activity relationship, and the structural design in chemistry, nanotechnology, and pharmacology.

The first molecular descriptor is the Wiener index, which was introduced by H. Wiener in 1947 in order to calculate the boiling points of paraffin [10]. Over the course of the last seventy years, many topological indices have been defined. These indices can be classified according to the structural characteristics of the graph such as the degree of vertices, the distances between vertices, the matching, and the spectrum and so on. The best-known topological indices are the Wiener index which is based on the distance, the Zagreb and the Randic indices which are based on degree, the Estrada index which is based on the spectrum of a graph, the Hosaya index which is based on thematching. Apart from these, it is a bond-additive index, which is a measure of peripherality in graphs.

Doslic et al. defined a new bond-additive topological index which is named Mostar index in 2019. In the same paper, they gave explicit formulas for benzenoid graph, Cartesian product, extremal and unicyclic graphs. Also, they stated several conjectures and open problems [3]. Tepeh proved their conjecture related with bicyclic graph [8].

In this study, the Mostar index which is the bond-additive index is studied. The edge Mostar index is defined. It is presented exact expressions for the Mostar index and edge Mostar index of wheel graph, gear graph, helm graph, friendship graph, and flower graph. These results are compared.

2. Preliminaries

Let G be a simple connected graph with a vertex set $V(G)$ and edge set $E(G)$ where $V(G) = \{v_1, v_2, .., v_n\}$. The number of a vertex set and edge set are defined by n and m, respectively. An edge of G connects the vertices u and v and it writes $e = uv$. The degree of a vertex u is defined by $d(u)$. The distance between vertices u and v is defined by $d(u, v)$. For standard terminology and notations we follow Buckley and Harary [2].

Mostar index is defined as

$$Mo(G) = \sum_{uv \in E(G)} |n_u - n_v| \tag{2.1}$$

where n_u is the number of vertices of G lying closer to vertex u than to vertex v of the edge uv [3]. Namely,

$$n_u = |N_u = \{x \in V(G) : d(x, u) < d(x, v)\}| . \tag{2.2}$$

Note that vertices equidistant to u and v not counted. Doslic et. al. presented following results [3]:

Corollary 2.1. *Let K_n be complete graph, C_n be cycle graph and $K_{n,n}$ be complete bipartite graph. Then, $Mo(K_n) = Mo(C_n) = Mo(K_{n,n}) = 0$.*

Corollary 2.2. *Let T_n be tree with n vertices and S_n be star graph with n vertices. Then, $Mo(T_n) \leq Mo(S_n) = (n-1)(n-2)$ with equality if only if $T_n = S_n$.*

Corollary 2.3. *Let P_n be path graph. Then, $Mo(P_n) = \left\lfloor \frac{(n-1)^2}{2} \right\rfloor$.*

The cycle graph related graphs are wheel graph, gear graph, helm graph, flower graph, and friendship graph.

Definition 2.4. *The wheel W_n for $n \geq 3$ is obtained by joining n-cycle and central vertex v_c. The wheel graph has $n+1$ vertices and $2n$ edges. The wheel graph consist of vertex set*

$$V(W_n) = V_1 \cup V_2$$

where

$$V_1 = \{v_i \in V(W_n) \,|\, d_{v_i} = 3, i = \overline{1, n}\}$$
$$V_2 = \{v_c \in V(W_n) \,|\, d_{v_c} = n\}$$

and edge set

$$E(W_n) = E_1 \cup E_2 \tag{2.3}$$

where

$$E_1 = \left\{v_i v_{i+1} \in E(W_n) \,\middle|\, v_i \in V_1, subscripts\ modula\ n, i = \overline{1, n}\right\},$$
$$E_2 = \left\{v_i v_c \in E(W_n) \,\middle|\, v_i \in V_1, i = \overline{1, n}\right\},$$

Definition 2.5. *Gear graph, G_n, is a wheel graph with a vertex added between each pair adjacent vertices of the outer cycle* [4], [1]. *The gear graph has $2n+1$ vertices and $3n$ edges. Obviously,*

$$V(G_n) = V_1 \cup V_2 \cup V_3$$

where

$$V_1 = \{v_i \in V(G_n) \,|\, d_{v_i} = 3, i = \overline{1, n}\},$$

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$$V_2 = \{u_i \in V(G_n) \,|\, d_{u_i} = 2, i = \overline{1,n}\},$$
$$V_3 = \{v_c \in V(G_n) \,|\, d_{v_c} = n\}$$

where v_c vertex is the center vertex of gear graph, V_1 vertex set is vertices of the outer cycle of wheel graph and V_2 is set of added vertices to the outer cycle. And edge set of G_n is

$$E(G_n) = E_1 \cup E_2 \cup E_3, \tag{2.4}$$

where

$$E_1 = \left\{v_i u_i \in E(G_n) \,\middle|\, v_i \in V_1, u_i \in V_2, i = \overline{1,n}\right\},$$
$$E_2 = \left\{u_i v_{i+1} \in E(G_n) \,\middle|\, \text{subscripts modula } n,\ v_{i+1} \in V_1, u_i \in V_2, i = \overline{1,n}\right\},$$
$$E_3 = \left\{v_i v_c \in E(G_n) \,\middle|\, v_i \in V_1, i = \overline{1,n}\right\}$$

and $|E_1| = n$, $|E_2| = n$, $|E_3| = n$.

Definition 2.6. *Helm graph H_n, is obtained from a wheel W_n with cycle C_n having a pendant edge attached to each vertex of cycle [4]. The helm graph has $2n+1$ vertices and $3n$ edges. Helm graph consists of*

$$V(H_n) = V_1 \cup V_2 \cup V_3$$

where

$$V_1 = \{v_i \in V(H_n) \,|\, d_{v_i} = 4, i = \overline{1,n}\}$$
$$V_2 = \{u_i \in V(H_n) \,|\, d_{u_i} = 1, i = \overline{1,n}\}$$
$$V_3 = \{v_c \in V(H_n) \,|\, d_{v_c} = n\}$$

where v_c vertex is the center vertex of helm graph, V_1 vertex set is vertices of the outer cycle of wheel graph and V_2 is set of added vertices to the wheel graph. Obviously,

$$E(H_n) = E_1 \cup E_2 \cup E_3, \tag{2.5}$$

where

$$E_1 = \left\{v_i v_{i+1} \in E(H_n) \,\middle|\, v_i \in V_1, \text{subscripts modula } n, i = \overline{1,n}\right\},$$
$$E_2 = \left\{v_i u_i \in E(H_n) \,\middle|\, v_i \in V_1\ , u_i \in V_2, i = \overline{1,n}\right\},$$
$$E_3 = \left\{v_i v_c \in E(H_n) \,\middle|\, v_i \in V_1, i = \overline{1,n}\right\}$$

and $|E_1| = n$, $|E_2| = n$, $|E_3| = n$.

Definition 2.7. *Friendship graph F_n, is obtained from a wheel W_{2n} with cycle C_{2n} by deleting alternate edges of the cycle [4]. The Friendship graph has $2n+1$ vertices and $3n$ edges. Friendship graph consists of*

$$V(F_n) = V_1 \cup V_2 \cup V_3$$

where

$$V_1 = \{v_i \in V(F_n) \,|\, d_{v_i} = 2, i = \overline{1,n}\},$$
$$V_2 = \{u_i \in V(F_n) \,|\, d_{u_i} = 2, i = \overline{1,n}\},$$
$$V_3 = \{v_c \in V(F_n) \,|\, d_{v_c} = 2n\}$$

where v_c vertex is the center vertex of friendship graph. Also, Friendship graph consists of

$$E(F_n) = E_1 \cup E_2 \cup E_3 \tag{2.6}$$

where

$$E_1 = \left\{v_i u_i \in E(F_n) \,\middle|\, v_i \in V_1, u_i \in V_2, i = \overline{1,n}\right\},$$
$$E_2 = \left\{v_i v_c \in E(F_n) \,\middle|\, v_i \in V_1, i = \overline{1,n}\right\},$$
$$E_3 = \left\{u_i v_c \in E(F_n) \,\middle|\, u_i \in V_2, i = \overline{1,n}\right\}$$

and $|E_1| = n$, $|E_2| = n$, $|E_3| = n$.

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Definition 2.8. *Flower graph Fl_n, is obtained from a wheel W_n by joining each pendant vertex to the central vertex and with cycle C_n having a pendant edge attached to each vertex of the outer cycle .The Flower graph has $2n+1$ vertices and $4n$ edges. Flower graph consists of*

$$V(Fl_n) = V_1 \cup V_2 \cup V_3$$

where

$$V_1 = \{v_i \in V(Fl_n) \mid d_{v_i} = 4, i = \overline{1,n}\},$$
$$V_2 = \{u_i \in V(Fl_n) \mid d_{u_i} = 2, i = \overline{1,n}\},$$
$$V_3 = \{v_c \in V(Fl_n) \mid d_{v_c} = 2n\}$$

where v_c vertex is the center vertex of the flower graph, V_1 vertex set is vertices of the outer cycle of wheel graph and V_2 is set of added vertices to the wheel graph. Obviously,

$$E(Fl_n) = E_1 \cup E_2 \cup E_3 \cup E_4, \tag{2.7}$$

where

$$E_1 = \left\{v_i v_{i+1} \in E(Fl_n) \,\middle|\, v_i \in V_1, \textit{subscripts modula } n, i = \overline{1,n}\right\},$$
$$E_2 = \left\{v_i v_c \in E(Fl_n) \,\middle|\, v_i \in V_1, i = \overline{1,n}\right\},$$
$$E_3 = \left\{u_i v_c \in E(Fl_n) \,\middle|\, u_i \in V_2, i = \overline{1,n}\right\},$$
$$E_4 = \left\{v_i u_i \in E(Fl_n) \,\middle|\, v_i \in V_1\ , u_i \in V_2, i = \overline{1,n}\right\},$$

and $|E_1| = n$, $|E_2| = n$, $|E_3| = n, |E_4| = n$.

3. Mostar Index of Some Cycle Related Graphs

In this section, it is given formulas for the mostar indices of gear, helm, flower and friendship graphs. Note that $d(v_i, v_i) = d(u_i, u_i) = 0$.

Theorem 3.1. *Mostar index of W_n wheel graph is*

$$Mo(W_n) = n\,(n-4)\,.$$

Proof. From Equations (2.1) and (2.3), we get

$$Mo(W_n) = \sum_{uv \in E_1} |n_u - n_v| + \sum_{uv \in E_2} |n_u - n_v|\,.$$

From the Definition 2.4, we can write following equalities for $i, j = \overline{1,n}$ and $i \neq j$:

$$d(v_i, x) = \begin{cases} 2, \text{ if } i, i-1, i+1 \neq j & x = v_j \\ 1, \text{ if } i-1, i+1 = j & x = v_j \\ 1, \text{ if } & x = v_c. \end{cases} \tag{3.1}$$

From Eq. (3.1), we can write the following cases:

Case 1. If $v_i v_{i+1} \in E(W_n)$, then it is obtained

$$n_{v_i} = \left|N_{v_i}\right| = |\{\{v_{i-1}\}, \{v_i\}\}| = 2,$$
$$n_{v_{i+1}} = \left|N_{v_{i+1}}\right| = |\{\{v_{i+1}\}, \{v_{i+2}\}\}| = 2.$$

Thus, we have:

$$\varepsilon_1 = \sum_{uv \in E_1} |n_u - n_v| = n\,|2-2| = 0.$$

Case 2. Let $v_i v_c \in E(W_n)$. We have

$$n_{v_i} = \left|N_{v_i}\right| = |\{v_i\}| = 1,$$
$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{v_{i-1}, v_i, v_{i+1}\}, v_c\}| = n - 3$$

Then, we have

$$\varepsilon_2 = \sum_{uv\in E_2} |n_{v_i} - n_{u_i}| = n\,|(n-3)-1| = n\,(n-4)\,.$$

By summing up the Cases 1 and 2, the proof is completed. □

Corollary 3.2. *Mostar index of W_{2n} wheel graph is*

$$Mo(W_{2n}) = 4n\,(n-2)\,.$$

Theorem 3.3. *Mostar index of G_n gear graph is*

$$Mo(G_n) = 3n(2n-5).$$

Proof. From Equations (2.1) and (2.4), we get

$$Mo(G_n) = \sum_{uv\in E_1} |n_u - n_v| + \sum_{uv\in E_2} |n_u - n_v| + \sum_{uv\in E_3} |n_u - n_v|\,.$$

From the Definition 2.5 , we can write following equalities for $i,j = \overline{1,n}$ and $i \neq j$:

$$d(v_i,x) = \begin{cases} 2, & x = v_j \\ 1, & x = v_c \end{cases}, \tag{3.2}$$

$$d(u_i,x) = \begin{cases} 3 & \text{for } \; i+1\neq j,\; x = v_j \\ 1 & \text{for } \; i+1 = j,\; x = v_j \\ 2 & \text{for } \; x = v_c \end{cases}, \tag{3.3}$$

$$d(u_i,u_j) = \begin{cases} 4\,, & \text{Otherwise} \\ 2\,, & i-1, i+1 = j \end{cases}, \tag{3.4}$$

Case 1. Let $v_iu_i \in E(G_n)$. From Equations (2.2), (3.2)- (3.4), we have

$$n_{v_i} = \left|N_{v_i}\right| = |\{V_1 - \{v_{i+1}\}, V_2 - \{u_i, u_{i+1}\}, v_c\}| = (n-1) + (n-2) + 1, \tag{3.5}$$

$$n_{u_i} = \left|N_{u_i}\right| = |\{v_{i+1}, u_i, u_{i+1}\}| = 3. \tag{3.6}$$

Thus, by Equations (3.5) and (3.6), we get

$$\varepsilon_1 = \sum_{uv\in E_1} |n_v - n_u| = n\,|2n-2-3| = n\,(2n-5)\,.$$

Case 2. Let $u_iv_{i+1} \in E(G_n)$. From Equations (2.2), (3.2)- (3.4), we have

$$n_{v_{i+1}} = \left|N_{v_{i+1}}\right| = |\{V_1 - \{v_i\}, V_2 - \{u_{i-1}, u_i\}, v_c\}| = (n-1) + (n-2) + 1, \tag{3.7}$$

$$n_{u_i} = \left|N_{u_i}\right| = |\{v_i, u_i, u_{i-1}\}| = 3. \tag{3.8}$$

By Equations (3.7) and (3.8), we get

$$\varepsilon_2 = \sum_{uv\in E_2} \left|n_{u_i} - n_{v_{i+1}}\right| = n\,|3-(2n-2)| = n\,(2n-5)\,.$$

Case 3. Let $v_iv_c \in E(G_n)$. From Equations (2.2), (3.2)- (3.4), we have

$$n_{v_i} = \left|N_{v_i}\right| = |\{v_i, u_{i-1}, u_i\}| = 3, \tag{3.9}$$

$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{v_i\}, V_2 - \{u_{i-1}, u_i\}, v_c\}| = (n-1) + (n-2) + 1. \tag{3.10}$$

By Equations (3.9) and (3.10), we get

$$\varepsilon_3 = \sum_{uv\in E_3} |n_{v_i} - n_{v_c}| = n\,|3-(2n-2)| = n\,(2n-5)\,.$$

By summing up the Cases 1, 2 and 3, it is clear that

$$Mo(G_n) = n(2n-5) + n(2n-5) + n(2n-5).$$

□

Theorem 3.4. *Mostar index of helm graph H_n with $n > 3$ is*

$$Mo(H_n) = 4n(n-2).$$

Proof. By Equations (2.1) and (2.5), we have:

$$Mo(H_n) = \sum_{uv\in E_1} |n_u - n_v| + \sum_{uv\in E_2} |n_u - n_v| + \sum_{uv\in E_3} |n_u - n_v| . \tag{3.11}$$

From the Definition 2.6, the following equations are written for $i,j = \overline{1,n}$

$$d(v_i,v_j) = \begin{cases} 2 \text{ for } & i-1,i,i+1 \neq j, x = v_j \\ 1 \text{ for } & i-1,i+1 = j, x = v_j \\ 1 \text{ for } & x = v_c \end{cases}, \tag{3.12}$$

$$d(v_i,u_j) = \begin{cases} 3 , & \text{Otherwise} \\ 2 , & i-1,i+1 = j \\ 1 , & j = i \end{cases}, \tag{3.13}$$

$$d(u_i,u_j) = \begin{cases} 4 \text{ for } & i-1,i,i+1 \neq j,\ x = u_j \\ 3 \text{ for } & i-1,i+1 = j,\ x = u_j \\ 2 \text{ for } & x = v_c \end{cases}. \tag{3.14}$$

From Equations (3.12)-(3.14), the following cases can be easily written:

Case 1. If $v_iv_{i+1} \in E(H_n)$, then it is obtained

$$n_{v_i} = \left|N_{v_i}\right| = \left|\{\{v_{i-1}\},\{v_i\},\{u_{i-1}\},\{u_i\}\}\right| = 4,$$

$$n_{v_{i+1}} = \left|N_{v_{i+1}}\right| = \left|\{\{v_{i+1}\},\{v_{i+2}\},\{u_{i+1}\},\{u_{i+2}\}\}\right| = 4.$$

Thus, we have:

$$\varepsilon_1 = \sum_{uv\in E_1} |n_u - n_v| = n\,|4-4| = 0.$$

Case 2. Let $v_iu_i \in E(H_n)$. We have

$$n_{v_i} = \left|N_{v_i}\right| = \left|\{V_1,\{v_c\},V_2-\{u_i\}\}\right| = 2n,$$

$$n_{u_i} = \left|N_{u_i}\right| = \left|\{u_i\}\right| = 1.$$

Then, we have

$$\varepsilon_2 = \sum_{uv\in E_2} |n_{v_i} - n_{u_i}| = n\,|2n-1| = n\,(2n-1)\,.$$

Case 3. Let $v_iv_c \in E(H_n)$. We have

$$n_{v_i} = \left|N_{v_i}\right| = \left|\{\{v_i\},\{u_i\}\}\right| = 2,$$

$$n_{v_c} = \left|N_{v_c}\right| = \left|\{V_1-\{v_{i-1},v_i,v_{i+1}\},\{v_c\},V_2-\{u_{i-1},u_i,u_{i+1}\}\}\right| = 2n-5.$$

Then, we have

$$\varepsilon_3 = \sum_{uv\in E_3} |n_{v_i} - n_{u_i}| = n\,|(2n-5)-2| = n\,(2n-7)\,.$$

By summing up the Cases 1, 2 and 3, the proof is completed. □

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Theorem 3.5. *Mostar index of friendship graph F_n is*

$$Mo(F_n) = 4n(n-1).$$

Proof. By Equations (2.1) and (2.6), we write

$$Mo(F_n) = \sum_{uv\in E_1} |n_u - n_v| + \sum_{uv\in E_2} |n_u - n_v| + \sum_{uv\in E_3} |n_u - n_v| . \tag{3.15}$$

From the Definition 2.7, we can write the following equations for $i, j = \overline{1, n}$

$$d(v_i, u_j) = \begin{cases} 2, & \text{Otherwise} \\ 1\,, & i = j \end{cases} , \tag{3.16}$$

$$d(v_i, x) = \begin{cases} 2 & \text{for } \; i \neq j, \; x = v_j \\ 1 & \text{for } \qquad x = v_c \end{cases} , \tag{3.17}$$

$$d(u_i, x) = \begin{cases} 2 & \text{for } \; i \neq j, x = u_j \\ 1 & \text{for } \quad x = v_c \end{cases} . \tag{3.18}$$

From Equations (3.16)-(3.18), the following cases are written

Case 1. Let $v_iu_i \in E(F_n)$.We easy see that

$$n_{v_i} = \left|N_{v_i}\right| = |\{v_i\}| = 1,$$

$$n_{u_i} = \left|N_{u_i}\right| = |\{u_i\}| = 1.$$

Then, we have

$$\varepsilon_1 = \sum_{uv\in E_1} |n_{v_i} - n_{u_i}| = n\,|1-1| = 0.$$

Case 2. Let $v_iv_c \in E(F_n)$. It is clear that

$$n_{v_i} = \left|N_{v_i}\right| = |\{v_i\}| = 1,$$

$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{v_i\}, \{v_c\}, V_2 - \{u_i\}\}| = 2n-1.$$

Then, we obtain

$$\varepsilon_2 = \sum_{uv\in E_2} |n_{v_i} - n_{v_c}| = n\,|1-(2n-1)| = n\,(2n-2)\,.$$

Case 3. Let $u_iv_c \in E(F_n)$. We easy see that

$$n_{u_i} = \left|N_{u_i}\right| = |\{u_i\}| = 1,$$

$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{v_i\}, \{v_c\}, V_2 - \{u_i\}\}| = 2n-1.$$

Then, we have

$$\varepsilon_3 = \sum_{uv\in E_3} |n_{u_i} - n_{v_c}| = n\,|1-(2n-1)| = n\,(2n-2)\,.$$

By summing up the Cases 1, 2 and 3, the proof is completed. □

Theorem 3.6. *Mostar index of flower graph Fl_n is*

$$Mo(Fl_n) = 4n(n-1).$$

Proof. By Equations 2.1 and 2.7, we write

$$Mo(Fl_n) = \sum_{uv\in E_1} |n_u - n_v| + \sum_{uv\in E_2} |n_u - n_v| + \sum_{uv\in E_3} |n_u - n_v| + \sum_{uv\in E_4} |n_u - n_v| .$$

From the Definition 2.8, we can write for $i, j = \overline{1, n}$

$$d(u_j, x) = \begin{cases} 2 \text{ for } i \neq j,\ x = v_i \\ 1 \text{ for } i = j,\ x = v_i \\ 1 \text{ for } \quad x = v_c \end{cases}, \tag{3.19}$$

$$d(v_i, x) = \begin{cases} 2 \text{ for } i-1, i+1 \neq j, & x = v_j \\ 1 \text{ for } i-1, i+1 = j, & x = v_j \\ 1 \text{ for } & x = v_c \end{cases}. \tag{3.20}$$

From Equations (3.19) and (3.20), we can write the following cases

Case 1. For $v_i v_{i+1} \in E(Fl_n)$, we have

$$n_{v_i} = \left|N_{v_i}\right| = |\{\{v_{i-1}\}, \{v_i\}, \{u_i\}\}| = 3,$$

$$n_{v_{i+1}} = \left|N_{v_{i+1}}\right| = |\{\{v_{i+1}\}, \{v_{i+2}\}, \{u_{i+1}\}\}| = 3,$$

Then, we obtain

$$\varepsilon_1 = \sum_{uv\in E_1} |n_{v_i} - n_{u_i}| = \sum_{uv\in E_1} |3 - 3| = 0.$$

Case 2. For $v_i v_c \in E(Fl_n)$, we have

$$n_{v_i} = \left|N_{v_i}\right| = |\{v_i\}| = 1,$$

$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{\{v_{i-1}\}, \{v_i\}, \{v_{i+1}\}\}, V_2 - \{u_i\}, \{v_c\}\}| = 2n - 3.$$

Then, it is easy see that

$$\varepsilon_2 = \sum_{uv\in E_2} |n_{v_i} - n_{v_c}| = \sum_{uv\in E_2} |1 - (2n - 3)| = n\,(2n - 4)\,.$$

Case 3. For $u_i v_c \in E(Fl_n)$, we have

$$n_{u_i} = |N_{u_i}| = |\{u_i\}| = 1,$$

$$n_{v_c} = \left|N_{v_c}\right| = |\{V_1 - \{v_i\}, V_2 - \{u_i\}, \{v_c\}\}| = 2n - 1.$$

Then, it is written the following equation

$$\varepsilon_3 = \sum_{uv\in E_3} |n_{v_i} - n_{u_i}| = \sum_{uv\in E_3} |1 - (2n - 1)| = n\,(2n - 2)\,.$$

Case 4. For $v_i u_i \in E(Fl_n)$, we have

$$n_{v_i} = |N_{v_i}| = |\{v_{i-1}, v_i, v_{i+1}\}| = 3,$$

$$n_{u_i} = |N_{u_i}| = |\{u_i\}| = 1.$$

Then, it is written the following equation

$$\varepsilon_4 = \sum_{uv\in E_4} |n_{v_i} - n_{u_i}| = \sum_{uv\in E_4} |3 - 1| = 2n.$$

By summing up the case 1, 2,3 and 4, the poof is completed. □

4. Edge Mostar Index of Some Cycle Related Graphs

In this section, the edge Mostar index is introduced. Then, the exact expressions for edge Mostar indices of gear, helm, flower and friendship graphs are given.

Motivated by the success results of [5], [6] the edge Mostar index is defined as

$$Mo_e(G) = \sum_{uv \in E(G)} |m_u - m_v| , \tag{4.1}$$

where m_u is the edge variants of the numbers n_u. That is, m_u is the number of edge of G lying closer to vertex u than to vertex v of the edge uv. That is, if the edges $e = uv$ and $f = xy$ of G, then

$$m_u = |d(u, f) < d(v, f)| , \tag{4.2}$$

where

$$d(u, f) = \min \{d(u, x), d(u, y)\} .$$

The edges $e = uv$ and $f = xy$ of G are said to be equidistant edges if $\min \{d(u, x), d(u, y)\} = min \{d(v, x), d(v, y)\}$. The equidistant edges are not counted.

Theorem 4.1. *$m_u = 0$ if and only if u is pendent vertex of G* [5].

Theorem 4.2. *In the case of trees, it is always the case that $m_u + m_v = n - 2 = m - 1$ and $m_u = n_u - 1$* [5].

Theorem 4.3. [7] *Let G be unicyclic graph and $e = uv \in E(C)$, where $E(C)$ is edge set of cycle.*

- **i:** For a unicyclic graphs with even girth, $n_u + n_v = n$, $m_u = n_u - 1$, $m_v = n_v - 1$ and $m_u + m_v = n - 2$.
- **ii:** Let a_i be the number of vertices of the component that contains the vertex c_i in $G - E(C)$. Then for a unicyclic graphs with odd girth, there exists a number a_i such that $n_u + n_v = n - a_i$, $m_u = n_u$, $m_v = n_v$ and $m_u + m_v = n - a_i$.

It is easily seen that the following theorem from Theorem 4.2, Corrollary 2.2 and Corrollary 2.3 :

Theorem 4.4. *If S_n is a star graph with order n, then*

$$Mo_e(S_n) = Mo(S_n) = m(m - 1) = (n - 1)(n - 2)$$

and

$$Mo_e(P_n) = Mo(P_n) = \left\lfloor \frac{(n - 1)^2}{2} \right\rfloor .$$

From Theorem 4.3 and Corrollary 2.1 , it is easy to obtain the following theorem:

Theorem 4.5. *If C_n is a cycle graph with n vertices, then $Mo_e(C_n) = Mo(C_n) = 0$.*

Theorem 4.6. *Mostar index of wheel graph W_n with $n > 3$ is*

$$Mo_e(W_n) = n(2n - 7) .$$

Proof. From Equations (4.1) and (2.3), we get

$$Mo(W_n) = \sum_{uv \in E_1} |m_u - m_v| + \sum_{uv \in E_2} |m_u - m_v| .$$

From Eq. (3.1), we can write the following cases

Case 1 Let $e = v_i v_{i+1} \in E(W_n)$.

i. If $f = v_i v_{i+1} \in E(W_n)$ then

$$m_{v_i} = |\{\{v_i v_{i-1}\}, \{v_{i-1} v_{i-2}\}\}| = 2,$$

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$m_{v_{i+1}} = |\{\{v_{i+1}v_{i+2}\}, \{v_{i+2}v_{i+3}\}\}| = 2.$
ii. If $f = v_iv_c \in E(W_n)$ then

$$m_{v_i} = |\{v_iv_c\}| = 1 \text{ and } m_{v_{i+1}} = |\{v_{i+1}v_c\}| = 1.$$

Thus, for $e = v_iv_{i+1} \in E(W_n)$ we have $\sum\limits_{uv\in E_1} |(2+1)-(2+1)| = 0.$

Case 2. Let $e = v_iv_c \in E(W_n)$.
i. If $f = v_iv_{i+1} \in E(W_n)$ then

$$m_{v_i} = |\{\{v_iv_{i-1}\}, \{v_iv_{i+1}\}\}| = 2,$$

$m_{v_c} = |\{E_1 - \{v_iv_{i-1}\}, \{v_iv_{i+1}\}, \{v_{i+1}v_{i+2}\}, \{v_{i-1}v_{i-2}\}\}| = n-4.$
ii. If $f = v_iv_c \in E(W_n)$ then

$$m_{v_i} = 0 \text{ and } m_{v_c} = |E_2 - \{v_iv_c\}| = n-1.$$

Thus, for $e = v_iv_c \in E(W_n)$ we have $\sum\limits_{uv\in E_2} |2-(2n-5)| = n(2n-7).$

By summiting up the cases 1 and 2, the proof is completed. □

Corollary 4.7. $Mo_e(W_{2n}) = 2n(4n-7).$

Theorem 4.8. *The edge Mostar index of gear graph is*

$$Mo_e(G_n) = 3n(3n-7).$$

Proof. From Eq. (4.1) and Eq. (2.4), we get

$$Mo(G_n) = \sum_{uv\in E_1} |m_u - m_v| + \sum_{uv\in E_2} |m_u - m_v| + \sum_{uv\in E_3} |m_u - m_v|.$$

By Equations (3.2)-(3.4) and (4.2), we easy can write the following cases:

Case 1. For $e = v_iu_i \in E(G_n)$.
i. If $f = v_ju_j \in E(G_n)$ then we obtain $m'_{v_i} = |E_1 - \{v_iu_i, v_{i+1}u_{i+1}\}|$ and $m'_{u_i} = |\{v_{i+1}u_{i+1}\}|.$
ii. If $f = u_jv_{j+1} \in E(G_n)$ then $m''_{v_i} = |E_2 - \{v_{i+1}u_i, v_{i+2}u_{i+1}\}|$ and $m''_{u_i} = |\{v_{i+1}u_i\}|.$
iii. If $f = v_iv_c \in E(G_n)$ then $m'''_{v_i} = |E_3 - \{v_{i+1}v_c\}|$ and $m'''_{u_i} = 0.$
By summiting up $m'_{v_i}, m''_{v_i}, m'''_{v_i}$ and $m'_{u_i}, m''_{u_i}, m'''_{u_i}$ for $v_iu_i \in E(G_n)$,we obtain

$$m_{v_i} = |E_1 + E_2 + E_3 - \{v_iu_i, v_{i+1}u_{i+1}, v_{i+2}u_{i+1}, v_{i+1}u_i, v_{i+1}v_c\}| = 3n-5$$

$$m_{u_i} = |\{v_{i+1}u_i, v_{i+1}u_{i+1}\}| = 2.$$

Thus, we have:

$$\varepsilon_1 = \sum_{uv\in E_1} |3n-5-2| = n(3n-7)$$

Case 2. For $e = u_iv_{i+1} \in E(G_n)$. We can write similarly way to Case 1:
i. If $f = v_ju_j \in E(G_n)$ then $m'_{u_i} = |\{v_iu_i\}|$ and $m'_{v_{i+1}} = |E_1 - \{v_iu_i, v_{i-1}u_{i-1}\}|.$
ii. If $f = u_jv_{j+1} \in E(G_n)$ then $m''_{u_i} = |\{v_iu_{i-1}\}|$ and $m''_{v_{i+1}} = |E_2 - \{v_iu_{i-1}, v_{i+1}u_i\}|.$
iii. If $f = v_iv_c \in E(G_n)$ then $m'''_{u_i} = 0$ and $m'''_{v_{i+1}} = |E_3 - \{v_iv_c\}|.$
By summiting up $m'_{v_{i+1}}, m''_{v_{i+1}}, m'''_{v_{i+1}}$ and $m'_{u_i}, m''_{u_i}, m'''_{u_i}$ for $u_iv_{i+1} \in E(G_n)$,we obtain

$$m_{v_{i+1}} = n-2+n-2+n-1 = 3n-5,$$

$$m_{u_i} = 1+1 = 2.$$

Thus, we have:

$$\varepsilon_2 = \sum_{uv\in E_2} |3n-5-2| = n(3n-7).$$

Case 3. For $e = v_iv_c \in E(G_n)$.
i. If $f = v_ju_j \in E(G_n)$ then $m'_{v_i} = |\{v_iu_i\}|$ and $m'_{v_c} = |E_1 - \{v_iu_i, v_{i-1}u_{i-1}\}|.$
ii. If $f = u_jv_{j+1} \in E(G_n)$ then $m''_{v_i} = |\{v_iu_{i-1}\}|$ and $m''_{v_c} = |E_2 - \{v_iu_{i-1}, v_{i+1}u_i\}|.$

iii. If $f = v_i v_c \in E(G_n)$ then $m'''_{v_i} = 0$ and $m'''_{v_c} = |E_3 - \{v_i v_c\}|$.

By summiting up $m'_{v_i}, m''_{v_i}, m'''_{v_i}$ and $m'_{v_c}, m''_{v_c}, m'''_{v_c}$ for $v_i v_c \in E(G_n)$,we obtain

$$m_{v_i} = n - 2 + n - 2 + n - 1 = 3n - 5,$$

$$m_{v_c} = 1 + 1 = 2.$$

Thus, we have:

$$\varepsilon_3 = \sum_{uv \in E_3} |3n - 5 - 2| = n(3n - 7).$$

By summiting up $\varepsilon_1, \varepsilon_2$ and ε_3, it is clear that $Mo_e(G_n) = n(3n - 7) + n(3n - 7) + n(3n - 7)$. □

Theorem 4.9. *The edge mostar index of helm graph with $n > 3$ is*

$$Mo_e(H_n) = 6n(n - 2).$$

Proof. From Eq. (4.1) and Eq. (2.5), we get

$$Mo_e(H_n) = \sum_{uv \in E_1} |m_u - m_v| + \sum_{uv \in E_2} |m_u - m_v| + \sum_{uv \in E_3} |m_u - m_v|.$$

By Equations (3.12)-(3.14) and (4.2), we easy can write the following cases:

Case 1. For $e = v_i v_{i+1} \in E(H_n)$.

i. If $f = v_j u_j \in E(H_n)$ then we have $m'_{v_i} = |\{v_i u_i, v_{i-1} u_{i-1}\}|$ and $m'_{v_{i+1}} = |\{v_{i+1} u_{i+1}, v_{i+2} u_{i+2}\}|$.

ii. If $f = v_j v_{j+1} \in E(H_n)$ then we have $m''_{v_i} = |\{v_i v_{i-1}, v_{i-1} v_{i-2}\}|$ and $m''_{v_{i+1}} = |\{v_{i+1} v_{i+2}, v_{i+2} v_{i+3}\}|$.

iii. If $f = v_j v_c \in E(H_n)$ then we have $m'''_{v_i} = |\{v_i v_c\}|$ and $m'''_{v_{i+1}} = |\{v_{i+1} v_c\}|$.

From i, ii and iii,

$m_{v_i} = |\{v_i u_i, v_{i-1} u_{i-1}, v_i v_{i-1}, v_{i-1} v_{i-2}, v_i v_c\}|$ and

$m_{v_{i+1}} = |\{v_{i+1} u_{i+1}, v_{i+2} u_{i+2}, v_{i+1} v_{i+2}, v_{i+2} v_{i+3}, v_{i+1} v_c\}|$ for $e = v_i v_{i+1} \in E(H_n)$. Thus,

$$\varepsilon_1 = \sum_{uv \in E_1} |5 - 5| = 0.$$

Case 2. For $e = v_i u_i \in E(H_n)$. From Theorem 4.1, we known that $m_{u_i} = 0$. And by Eq. (4.2), we have $m_{v_i} = |E_1 - \{v_i u_i\}, E_2, E_3| = 3n - 1$. Then, we have

$$\varepsilon_2 = \sum_{uv \in E_2} |m_u - m_v| = n(3n - 1).$$

Case 3. For $e = v_i v_c \in E(H_n)$.

i. If $f = v_j u_j \in E(H_n)$ then we have $m'_{v_i} = |\{v_i u_i\}| = 1$ and $m'_{v_c} = |E_1 - \{v_{i-1} u_{i-1}, v_i u_i, v_{i+1} u_{i+1}\}|$.

ii. If $f = v_j v_{j+1} \in E(H_n)$ then we have $m''_{v_i} = |\{v_i v_{i-1}, v_i v_{i+1}\}|$ and

$m''_{v_c} = |E_2 - \{v_i v_{i-1}, v_i v_{i+1}, v_{i+1} v_{i+2}, v_{i-1} v_{i-2}\}| = n - 4$.

iii. Let $f = v_j v_c \in E(H_n)$. we have $m'''_{v_i} = 0$ and $m'''_{v_c} = |E_3 - \{v_i v_c\}|$.

Thus, for $e = v_i v_c \in E(H_n)$,we have:

$$\varepsilon_3 = \sum_{uv \in E_3} |(1 + 2) - ((n - 3) + (n - 4) + (n - 1))| = n(3n - 11).$$

From summing up ε_1, ε_2 and ε_3, it is obtained that $Mo_e(H_n) = n(3n-1)+n(3n-11) = n(6n-12)$. □

Theorem 4.10. *The edge mostar index of flower graph is*

$$Mo_e(F_n) = 2n(4n - 5).$$

Proof. From Eq. (4.1) and Eq. (2.7), we write

$$Mo_e(F_n) = \sum_{uv \in E_1} |m_u - m_v| + \sum_{uv \in E_2} |m_u - m_v| + \sum_{uv \in E_3} |m_u - m_v| + \sum_{uv \in E_4} |m_u - m_v|.$$

By Equations (3.19)-(3.20) and (4.2), we easy can write the following cases:

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Case 1. For $e = v_iv_{i+1} \in E(F_n)$.

i. If $f = v_ju_j \in E(F_n)$ then we have $m'_{v_i} = |\{u_{i-1}v_{i-1}, u_iv_i\}| = 2$ and $m'_{v_{i+1}} = |\{u_{i+1}v_{i+1}, u_{i+2}v_{i+2}\}| = 2$.

ii. If $f = v_jv_{j+1} \in E(F_n)$ then we easy see that $m''_{v_i} = |\{v_{i-1}v_{i-2}, v_{i-1}v_i\}| = 2$ and $m''_{v_{i+1}} = |\{v_{i+1}v_{i+2}, v_{i+2}v_{i+3}\}| = 2$.

iii. If $f = v_jv_c \in E(F_n)$, then we have $m'''_{v_i} = |\{v_iv_c\}| = 1$ and $m'''_{v_{i+1}} = |\{v_{i+1}v_c\}| = 1$.

iv. Let $f = u_jv_c \in E(F_n)$.we have $m''''_{v_i} = 0$ and $m''''_{v_{i+1}} = 0$.

Thus, from i,ii,iii and iv for $v_iv_{i+1} \in E(F_n)$, we have

$$\varepsilon_1 = \sum_{uv \in E_1} |(2+2+1) - (2+2+1)| = 0.$$

Case 2. For $e = v_iv_c \in E(F_n)$.

i. If $f = v_ju_j \in E(F_n)$ then we have $m'_{v_i} = |\{u_iv_i\}| = 1$ and $m'_{v_c} = |E_1 - \{u_{i-1}v_{i-1}, u_iv_i, u_{i+1}v_{i+1}\}|$.

ii. If $f = v_jv_{j+1} \in E(F_n)$ then we easy see that $m''_{v_i} = |\{v_iv_{i+1}, v_{i-1}v_i\}| = 2$ and $m''_{v_c} = |E_2 - \{v_{i-1}v_{i-2}, v_{i-1}v_i, v_iv_{i+1}, v_{i+1}v_{i+2}\}|$.

iii. If $f = v_jv_c \in E(F_n)$ then we have $m'''_{v_i} = 0$ and $m'''_{v_c} = |E_3 - \{v_iv_c\}|$ because of equidistant edges.

iv. If $f = u_jv_c \in E(F_n)$ then $m''''_{v_i} = 0$ and $m''''_{v_c} = n$.

Thus, we have for $v_iv_c \in E(F_n)$,

$$\varepsilon_2 = \sum_{uv \in E_2} |(1+2) - ((n-3) + (n-4) + (n-1) + n)| = n\,(4n-11)\,.$$

Case 3. For $e = u_iv_c \in E(F_n)$.

i. If $f = v_ju_j \in E(F_n)$ then we have $m'_{u_i} = |\{u_iv_i\}| = 1$ and $m'_{v_c} = |E_1 - \{u_iv_i\}|$.

ii. If $f = v_jv_{j+1} \in E(F_n)$ then we easy see that $m''_{u_i} = 0$ and $m''_{v_c} = |E_2 - \{v_{i-1}v_i, v_iv_{i+1}\}|$.

iii. If $f = v_jv_c \in E(F_n)$ then because of equidistant edges, we have $m'''_{u_i} = 0$ and $m'''_{v_c} = |E_3|$.

iv. If $f = u_jv_c \in E(F_n)$ then we have $m''''_{u_i} = 0$ and $m''''_{v_c} = |E_4 - \{u_iv_c\}|$.

Thus, for $u_iv_c \in E(F_n)$, we have

$$\varepsilon_3 = \sum_{uv \in E_3} |1 - ((n-1) + (n-2) + n + (n-1))| = n\,(4n-5)\,.$$

By summing up ε_1, ε_2, ε_3 and ε_4, the proof is completed.

Case 4. For $e = v_iu_i \in E(F_n)$.

i. If $f = v_ju_j \in E(F_n)$ then we have $m'_{v_i} = |\{u_{i-1}v_{i-1}, u_{i+1}v_{i+1}\}| = 2$ and $m'_{u_i} = 0$.

ii. If $f = v_jv_{j+1} \in E(F_n)$ then we easy see that $m''_{v_i} = |\{v_{i-1}v_{i-2}, v_{i-1}v_i, v_{i+1}v_{i+2}, v_iv_{i+1}\}| = 4$ and $m''_{u_i} = 0$.

iii. If $f = v_jv_c \in E(F_n)$ then we have $m'''_{v_i} = |\{v_iv_c\}|$ and $m'''_{u_i} = 0$ because of the equidistant edges.

iv. If $f = u_jv_c \in E(F_n)$ then we have $m''''_{v_i} = 0$ and $m''''_{u_i} = |\{u_iv_c\}| = 1$.

Thus we have from i, ii, iii, iv for $v_iu_i \in E(F_n)$

$$\varepsilon_4 = \sum_{uv \in E_4} |(2+4+1) - 1| = 6n.$$

□

Theorem 4.11. *The edge mostar index of friendship graph is*

$$Mo_e(Fl_n) = 6n(n-1).$$

Proof. From Eq. (4.1) and Eq. (2.6), we get

$$Mo_e(Fl_n) = \sum_{uv \in E_1} |m_u - m_v| + \sum_{uv \in E_2} |m_u - m_v| + \sum_{uv \in E_3} |m_u - m_v|\,.$$

By Equations (3.16)-(3.18) and (4.2), we easy can write the following cases:

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Case1. For $e = v_i u_i \in E(Fl_n)$.
i. Let $f = v_j u_j \in E(Fl_n)$.We have $m_{v_i} = 0$ and $m_{u_i} = 0$ because all edges are equidistant edges.
ii. Let $f = v_j v_c \in E(Fl_n)$. We have $m_{v_i} = |\{v_i v_c\}| = 1$ and $m_{u_i} = 0$.
iii. Let $f = u_j v_c \in E(Fl_n)$. We have $m_{v_i} = 0$ and $m_{u_i} = |\{u_i v_c\}| = 1$.
Thus, we obtain from i, ii, iii for $v_i u_i \in E(Fl_n)$:

$$\varepsilon_1 = \sum_{uv \in E_1} |1 - 1| = 0.$$

Case2. For $e = v_i v_c \in E(Fl_n)$.
i. Let $f = v_j u_j \in E(Fl_n)$.We have $m_{v_i} = |\{u_i v_i\}| = 1$ and $m_{v_c} = |E_1 - \{u_i v_i\}|$
ii. Let $f = v_j v_c \in E(Fl_n)$. We have $m_{v_i} = 0$ and $m_{v_c} = |E_2 - \{v_i v_c\}|$
iii. Let $f = u_j v_c \in E(Fl_n)$. We have $m_{v_i} = 0$ and $m_{v_c} = |E_3|$.
Thus, we obtain from i, ii, iii for $v_i v_c \in E(Fl_n)$:

$$\varepsilon_2 = \sum_{uv \in E_2} |1 - ((n-1) + (n-1) + n)| = n\,(3n-3)\,.$$

Case3. For $e = u_i v_c \in E(Fl_n)$.
i. Let $f = v_j u_j \in E(Fl_n)$.We have $m_{u_i} = |\{u_i v_i\}|$ and $m_{v_c} = |E_1 - \{u_i v_i\}|$.
ii. Let $f = v_j v_c \in E(Fl_n)$. We have $m_{u_i} = 0$ and $m_{v_c} = |E_2|$.
iii. Let $f = u_j v_c \in E(Fl_n)$. We have $m_{u_i} = 0$ and $m_{v_c} = |E_3 - \{u_i v_c\}|$.
Thus, we obtain from i, ii, iii for $u_i v_c \in E(Fl_n)$:

$$\varepsilon_3 = \sum_{uv \in E_3} |1 - ((n-1) + n + (n-1))| = n\,(3n-3)\,.$$

From summing up ε_1, ε_2 and ε_3, the proof is completed. □

5. Compare of Mostar index (Mo) and Edge Mo index for Some Cycle Related Graphs

In this section, we compare of the Mostar index (Mo) and the edge Mo index for some cycle related graphs which are the wheel graph, the gear graph, the helm graph, the friendship graph, and the flower graph. These considered graphs have the same order and have the same size without Fl_n and W_{2n}. Thus, we can make these comparisons. Figure 1 shows the Mo index value of considered graphs and also the edge Mo index theirs is depicted in Figure 2. The Mo index values of F_n ile Fl_n and also H_n ile W_{2n} are same but the edge Mostar index values of considered graphs are not the same.

From Figure 1, we see that Mo index of G_n is better than Mo indices of $H_n(W_{2n})$ and $F_n(Fl_n)$ and also Mo index value of F_n is better than H_n but these are very close.

From Figure 2, we see that Mo_e index values of considered graphs are nearly the same. The edge Mostar index is based on edge distance. So, the same number of edges making comparisons would be more correct to compare with each other. The size of Fl_n with W_{2n} are the same and from figure 2, we can say Fl_n is better than W_{2n}. And G_n is better than H_n and F_n.

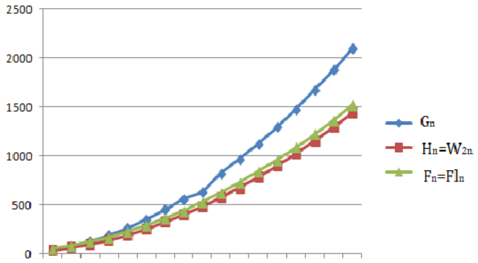

Figure 1. Mo index of Some Cycle Related Graphs

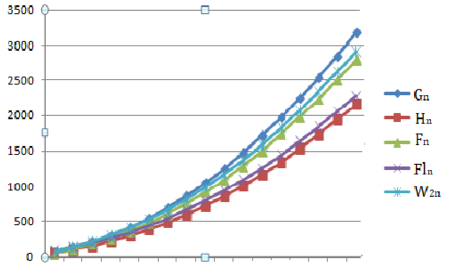

Figure 2. Mo_e index of Some Cycle Related Graphs

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