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RECURRENT TRANS-SASAKIAN MANIFOLDS

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Abstract. A Riemannian manifold is said to be recurrent if the Riemannian curvature tensor R is recurrent. On the other hand, A. Oubina defined a new class of almost contact metric manifolds, named trans-Sasakian manifolds, which is a generalzation of Sasakian and Kenmotsu manifolds. In this paper, first we define the notion of a recurrent trans-Sasakian manifold. Then we give an information on the recurrent form on a recurrent trans-Sasakian manifold. Using this result, we show that the Riemannian curvature tensor, the Ricci tensor and the scalar curvature on a recurrent trans-Sasakian manifold are expressed in terms of the associated functions. Finally, we show that the recurrent form of a recurrent trans-Sasakian manifold is written by the associated functions.

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1. Trans-Sasakian Manifolds

Let M be a real $(2n+1)$-dimensional almost contact metric manifold with structure (φ, ξ, η, g), satisfying

$$\begin{cases} \varphi^2 = -I + \eta \otimes \xi, \ \ \eta\varphi = \varphi\xi = 0, \ \ g(X, \xi) = \eta(X), \\ g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y). \end{cases} \tag{1.1}$$

Definition 1.1. An almost contact metric manifold M with the structure (φ, ξ, η, g) is said to be a *trans-Sasakian manifodd of* (α, β)*-type* if it satisfies

$$(\nabla_X \varphi)Y = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\varphi X, Y)\xi - \eta(Y)\varphi X\}, \tag{1.2}$$

for certain functions α and β on M, which are called *associated functions*, where ∇ means the covariant differentiation with respect to g.

Remark 1.2. On a trans-Sasakian manifold M, if $\alpha = 0$ (resp. $\beta = 0$), we say M is a β-Kenmotsu (resp. α-Sasakian) manifold.

Remark 1.3. In [3], Oubina gave an example of 3-dimensional trans-Sasakian manifold which is neither Sasakian nor Kenmotsu.

We know the following formulae on a trans-Sasakian manifold M of (α,β)-type.

About the structure vector field ξ and 1-form η, we can easily get from (1.1) and (1.2)

$$\begin{cases}\nabla_X\xi = -\alpha\varphi X + \beta\{X - \eta(X)\xi\},\\ (\nabla_Y\eta)(X) = -\alpha g(\varphi Y, X) + \beta\{g(Y,X) - \eta(Y)\eta(X)\}.\end{cases} \tag{1.3}$$

Next, about the Riemannian curvature tensor R, we have from (1.3)

$$\begin{aligned} R(X,Y,Z,\xi) &= (X\alpha)g(\varphi Y,Z) - (Y\alpha)g(\varphi X,Z)\\ &-(X\beta)\{g(Y,Z) - \eta(Y)\eta(Z)\} + (Y\beta)\{g(X,Z) - \eta(X)\eta(Z)\}\\ &+(\alpha^2-\beta^2)A(X,Y,Z) - 2\alpha\beta B(X,Y,Z), \end{aligned} \tag{1.4}$$

where we put

$$A(X,Y,Z) = g(Z,Y)X - g(Z,X)Y, \quad B(X,Y,Z) = A(X,Y,\varphi Z). \tag{1.5}$$

From (1.4), we can easily obtain

$$\rho(X,\xi) = -(X\tilde{\alpha}) - (2n-1)(X\beta) - \{\xi(\beta) - 2n(\alpha^2-\beta^2)\}\eta(X), \tag{1.6}$$

where ρ means the Ricci tensor with respect to g.

2. Covariant differentiations of the Riemannian curvature tensor, the Ricci tensor and the scalar curvature

By the covariant derivation of (1.4), we get

$$\begin{aligned} &-\alpha R(X,Y,Z,\varphi W) + \beta R(X,Y,Z,W)\\ &= -(\nabla_W R)(X,Y,Z,\xi) + \beta[(X\alpha)g(\varphi Y,Z) - (Y\alpha)g(\varphi X,Z)\\ &-(X\beta)\{g(Y,Z) - \eta(Y)\eta(Z)\} + (Y\beta)\{g(X,Z) - \eta(X)\eta(Z)\}\\ &+(\alpha^2-\beta^2)A(X,Y,Z) - 2\alpha\beta B(X,Y,Z)\}\eta(W)\\ &+(\nabla_W\alpha)(X)g(\varphi Y,Z) - (\nabla_W\alpha)(Y)g(\varphi X,Z)\\ &-(\nabla_W\beta)(X)\{g(Y,Z) - \eta(Y)\eta(Z)\} + (\nabla_W\beta)(Y)\{g(X,Z) - \eta(X)\eta(Z)\}\\ &+2\{\alpha(W\alpha) - \beta(W\beta)\}A(X,Y,Z) - 2\{\beta(W\alpha) + \alpha(W\beta)\}B(X,Y,Z)\\ &-(X\alpha)\{\alpha A(Y,Z,W) + \beta B(Y,Z,W)\} + (Y\alpha)\{\alpha A(X,Z,W) + \beta B(X,Z,W)\}\\ &+(X\beta)[\alpha\{g(\varphi Y,W)\eta(Z) + g(\varphi Z,W)\eta(Y)\} + \beta\{g(Y,W)\eta(Z) + g(W,Z)\eta(Y)\\ &-2\eta(Y)\eta(Z)\eta(W)\}] - (Y\beta)[\alpha\{g(\varphi X,W)\eta(Z) + g(\varphi Z,W)\eta(X)\}\\ &+\beta\{g(X,W)\eta(Z) + g(W,Z)\eta(X) - 2\eta(X)\eta(Z)\eta(W)\}]\\ &+(\alpha^2-\beta^2)[\alpha\{g(Y,Z)g(\varphi X,W) - g(X,Z)g(\varphi Y,W)\}\\ &+\beta\{g(X,W)g(Y,Z) - g(X,Z)g(Y,W) - A(X,Y,Z)\eta(W)\}]\\ &+2\alpha\beta[\alpha\{g(\varphi X,W)g(\varphi Y,Z) - g(\varphi X,Z)g(\varphi Y,X) + A(X,Y,W)\eta(Z)\}\\ &+\beta\{g(W,Y)g(\varphi X,Z) - g(X,W)g(\varphi Y,Z) - B(X,Y,W)\eta(Z) - B(X,Y,Z)\eta(W)\}]. \end{aligned} \tag{2.1}$$

By virtue of the above equation, we obtain

$$(\alpha^2+\beta^2)R(X,Y,Z,W) = -\beta(\nabla_W R)(X,Y,Z,\xi) \tag{2.2}$$

$$-\alpha(\nabla_{\varphi W}R)(X,Y,Z,\xi)+\{\beta(\nabla_W\alpha)(X)+\alpha(\nabla_{\varphi W}\alpha)(X)\}g(\varphi Y,Z)$$
$$-\{\beta(\nabla_W\alpha)(Y)+\alpha(\nabla_{\varphi W}\alpha)(Y)\}g(\varphi X,Z)$$
$$+\{\beta(\nabla_W\beta)(X)-\alpha(\nabla_{\varphi W}\beta)(X)\}\{g(Y,Z)-\eta(Y)\eta(Z)\}$$
$$-\{\beta(\nabla_W\beta)(Y)-\alpha(\nabla_{\varphi W}\beta)(Y)\}\{g(X,Z)-\eta(X)\eta(Z)\}$$
$$+2\{\alpha\beta(W\alpha)-\beta^2(W\beta)+\alpha^2(W\tilde{\alpha})-\alpha\beta(W\tilde{\beta})\}A(X,Y,Z)$$
$$-2\{\beta^2(W\alpha)+\alpha\beta(W\beta)+\alpha\beta(W\tilde{\alpha})+\alpha^2(W\tilde{\beta})\}B(X,Y,Z)$$
$$+(\alpha^2+\beta^2)[(X\alpha)\{g(\varphi Y,Z)\eta(W)-B(Y,Z,W)\}$$
$$-(Y\alpha)\{g(\varphi X,Z)\eta(W)-B(X,Z,W)\}$$
$$+(X\beta)\{g(Y,W)\eta(Z)-A(Y,W,Z)-\eta(Y)\eta(Z)\eta(W)\}$$
$$-(Y\beta)\{g(Z,W)\eta(X)+A(Z,W,X)-\eta(Z)\eta(W)\eta(X)\}$$
$$+(\alpha^2-\beta^2)\{g(X,W)g(Y,Z)-g(X,Z)g(Y,W)\}$$
$$+2\alpha\beta\{g(X,W)g(\varphi Y,Z)-g(Y,W)g(\varphi,Z)+B(X,Y,W)\eta(Z)\}].$$

From (1.6), for the Ricci tensor ρ, we have locally

$$\beta\rho_{ji}-\alpha\rho_{il}\varphi_j{}^l=-(\nabla_j\rho_{il})\xi^l-(2n-1)(\nabla_j\beta_i)-\xi^l(\nabla_j\beta_l)\eta_i-\varphi_i{}^l(\nabla_j\alpha_l)$$
$$+4n(\alpha\alpha_j-\beta\beta_j)\eta_i-\beta\{\tilde{\alpha}_i+(2n-1)\beta_i\}\eta_j+(\alpha\alpha_j+\beta\tilde{\alpha}_j)\eta_i$$
$$+[\alpha\tilde{\beta}_j-\beta\{\beta_j-\xi(\beta)\eta_j\}]\eta_i-[\alpha\xi(\alpha)+\beta\{\xi(\beta)-2n(\alpha^2-\beta^2)\}]g_{ji}$$
$$-[\beta\xi(\alpha)-\alpha\{\xi(\beta)-2n(\alpha^2-\beta^2)\}]\varphi_{ji}. \tag{2.3}$$

Using (2.3), we write

$$(\alpha^2+\beta^2)\rho_{ji}=-\beta(\nabla_j\rho_{il})\xi^l-\alpha(\nabla_m\rho_{il})\varphi_j{}^m\xi^l-\beta Q_{ji}-\alpha\tilde{Q}_{ji}$$
$$+4n(\alpha\beta\alpha_j-\beta^2\beta_j+\alpha^2\tilde{\alpha}_j-\alpha\beta\tilde{\beta}_j)\eta_i-(\alpha^2+\beta^2)[\{\tilde{\alpha}_i+(2n-1)\beta_i\}\eta_j$$
$$+\{\beta_j-\xi(\beta)\eta_j\}\eta_i-\tilde{\alpha}_j\eta_i+\{\xi(\beta)-2n(\alpha^2-\beta^2)\}g_{ji}+\xi(\alpha)\varphi_{ji}], \tag{2.4}$$

where we put

$$Q_{ji}=\varphi_i{}^l(\nabla_j\alpha_l)+(2n-1)(\nabla_j\beta_i)-\xi^l(\nabla_j\beta_l)\eta_i \tag{2.5}$$

and $\tilde{Q}_{ji}=Q_{mi}\varphi_j{}^m$.

Moreover, using (2.4) and the Bianchi identity [4], the scalar curvature τ is written by

$$(\alpha^2+\beta^2)\tau=\frac{1}{2}\beta(\nabla_\tau)\xi^l-\alpha(\nabla_m\rho_{il})\varphi^{im}\xi^l-\beta\{(2n-1)(\delta\beta)+\xi^l\xi^m(\nabla_m\beta_l)\}$$
$$-\alpha\{(\delta\alpha)-\xi^m\xi^l(\nabla_m\alpha_l)\}+4n\{\alpha\beta\xi(\alpha)-\beta^2\xi(\beta)\}$$
$$-2n(\alpha^2+\beta^2)\{2\xi(\beta)-(2n+1)(\alpha^2-\beta^2)\}, \tag{2.6}$$

where $\delta\alpha$ (resp. $\delta\beta$) means the codifferential of α (resp. β).

3. Recurrent trans-Sasakian manifolds

Definition 3.1. On a Riemannian manifold, a tensor field T is called *recurrent* if it satisfies

$$\nabla_X T = \Pi(X)T, \tag{3.1}$$

for any tangent vector field X and a certain 1-form Π, which is called a *recurrent form* of T.

Definition 3.2. A Riemannian manifold is said to be *recurrent* if its Riemannian curvature tensor R is recurrent.

So, a *recurrent Riemannian manifold* means that its Riemannian curvature is recurrent.

In this section, we assume that a trans-Sasakian manifold is recurrent with a recurrent form Π (we call it a *recurrent trans-Sasakian manifold*), that is, the Riemannian curvature tensor R satisfies

$$(\nabla_U R)(X,Y,Z,W) = \Pi(U)R(X,Y,Z,W), \tag{3.2}$$

for a certain 1-form Π.

From now on, we always assume that the associated functions α and β satisfy $\alpha\beta \neq 0$

We can easily see that the recurrent Riemannian manifold has the Ricci recurrence with the same recurrent form, that is,

$$(\nabla_Z \rho)(X,Y) = \Pi(Z)\rho(X,Y). \tag{3.3}$$

From (2.4) and (1.6), the Ricci tensor ρ satisfies

$$\begin{aligned}(\alpha^2+\beta^2)\rho_{ji} &= -(\beta\Pi_j + \alpha\tilde{\Pi}_j)[\tilde{\alpha}_i + (2n-1)\beta_i + \{\xi(\beta) - 2n(\alpha^2-\beta^2)\}\eta_i] \\ &\quad -(\beta Q_{ji} + \alpha\tilde{Q}_{ji}) + 4n(\alpha\beta\alpha_j - \beta^2\beta_j + \alpha^2\tilde{\alpha}_j - \alpha\beta\tilde{\beta}j)\eta_i \\ &\quad -(\alpha^2+\beta^2)[\{\tilde{\alpha}_i + (2n-1)\beta_i\}\eta_j + \{\beta_j - \xi(\beta)\eta_j\}\eta_i - \tilde{\alpha}_j\eta_i \\ &\quad +\{\xi(\beta) - 2n(\alpha^2-\beta^2)\}g_{ji} + \xi(\alpha)\varphi_{ji}].\end{aligned} \tag{3.4}$$

Contraction of (3.4) by ξ^i and (1.6) gives us

$$\begin{aligned}n\{\xi(\beta) - (\alpha^2-\beta^2)\}(\beta\Pi_j + \alpha\tilde{\Pi}_j) &= (\alpha^2+\beta^2)[\tilde{\alpha}_j + (n-1)\{\beta_j - \xi(\beta)\eta_j\}] \\ &\quad +(n-1)\{\beta(\nabla_j\beta_l) + \alpha\varphi_j{}^m(\nabla_m\beta_l)\}\xi^l + 2n(\alpha\beta\alpha_j - \beta^2\beta_j + \alpha^2\tilde{\alpha}_j - \alpha\beta\tilde{\beta}_j).\end{aligned} \tag{3.5}$$

From (3.4) and (3.5) we get

$$\begin{aligned}n\{\xi(\beta) - (\alpha^2-\beta^2)\}(\alpha^2+\beta^2)\rho_{ji} &= -\big[(\alpha^2+\beta^2)[\tilde{\alpha}_j + (n-1)\{\beta_j - \xi(\beta)\eta_j\} \\ &\quad +(n-1)\{\beta(\nabla_j\beta_l) + \alpha\varphi_j{}^m(\nabla_m\beta_l)\}\xi^l - 2n(\alpha\beta\alpha_j - \beta^2\beta_j + \alpha^2\tilde{\alpha}_j - \alpha\beta\tilde{\beta}_j)\big][\tilde{\alpha}_i \\ &\quad +(2n-1)\beta_i + \{\xi(\beta) - 2n(\alpha^2-\beta^2)\}\eta_i] - n\{\xi(\beta) - (\alpha^2-\beta^2)\}(\beta Q_{ji} + \alpha\tilde{Q}_{ji}) \\ &\quad -n\{\xi(\beta) - (\alpha^2-\beta^2\}(\alpha^2+\beta^2)[\{\tilde{\alpha}_i + (n2n-1)\beta_i\}\eta_i + \{\beta_j - \xi(\beta)\eta_j\}\eta_i \\ &\quad -\tilde{\alpha}_j\eta_i + \{\xi(\beta) - 2n(\alpha^2-\beta^2)\}g_{ji} + \xi(\alpha)\varphi_{ji}].\end{aligned} \tag{3.6}$$

From (3.6), the scalar curvature τ is given by

$$\begin{aligned}n\{\xi(\beta) - (\alpha^2-\beta^2)\}(\alpha^2+\beta^2)\tau &= -4n(n-1)\alpha\beta(\beta^i\alpha_i) \\ &\quad -n\{(4n+1)\alpha^2 + (\beta+2)\beta^2\}(\beta^i\tilde{\alpha}_i) - \{(n+1)\alpha^2+\beta^2\}\|\alpha\|^2 \\ &\quad +(2n-1)\{(n-1)\alpha^2 + (3n-1)\beta^2\}\|\beta\|^2 + \{(2n+1)\alpha^2+\beta^2\}\xi^2(\alpha) \\ &\quad -(2n-1)(n-1)(\alpha^2+\alpha^2)\xi^2(\beta) + \alpha\{\xi(\beta) - (\alpha^2-\beta^2)\}\xi^m\xi^l(\nabla_m\alpha_l) \\ &\quad -n\{\alpha\xi(\alpha) - 2\beta\xi(\beta) + (2n+1)\beta(\alpha^2-\beta^2)\}\xi^m\xi^l(\nabla_m\beta_l)\end{aligned} \tag{3.7}$$

$$+n\{\beta(\nabla^m\beta_l)\tilde{\alpha}_m+\alpha(\nabla^m\beta_l)\alpha_m\}\xi^l+n(n-1)\{\frac{1}{2}\beta\nabla\|\beta\|^2+\alpha\tilde{\beta}^m(\nabla_m\beta_l)\}\xi^l$$
$$-2n\alpha\beta\xi(\alpha)\xi(\beta)-2n\beta\{\xi(\beta)-2h(\alpha^2-\beta^2)\}(\alpha\xi(\alpha)-\beta\xi(\beta)\}$$
$$-\{\xi(\beta)-(\alpha^2-\beta^2)\{\alpha(\delta\alpha)+(2n-1)\beta(\delta\beta)\}.$$

Next, on a recurrent trans-Sasakian manifold, the curvature tensor is written by

$$\begin{aligned}(\alpha^2+\beta^2)R_{kjih}&=-(\beta\Pi_k+\alpha\tilde{\Pi}_k)\{\alpha_j\varphi_{ih}-\alpha_i\varphi_{jh}-\beta_j(g_{ih}-\eta_i\eta_h)\\&+\beta_i(g_{jh}-\eta_j\eta_h)+(\alpha^2-\beta^2)A_{jih}-2\alpha\beta B_{jih}\}+\beta T_{kjih}+\alpha T_{mjih}\varphi_k{}^m\\&+2(\alpha\beta\alpha_k-\beta^2\beta_k+\alpha^2\tilde{\alpha}_k-\alpha\beta\tilde{\beta}_k)A_{jih}+2(\alpha^2\alpha_k+\alpha\beta\beta_k+\alpha\beta\tilde{\alpha}_k+\beta^2\tilde{\beta}_k)B_{jih}\\&+(\alpha^2+\beta^2)[\{\alpha_j\varphi_{ih}-\alpha_i\varphi_{jh}-\beta_j(g_{ih}-\eta_i\eta_h)+\beta_i(g_{jh}-\eta_j\eta_h)+(\alpha^2-\beta^2)A_{jih}\\&-2\alpha\beta B_{jih}\}\eta_k+\alpha_jB_{kih}-\alpha_iB_{hjk}+\beta_j(g_{ki}\eta_h+g_{ki}\eta_i-2\eta_k\eta_i\eta_h)\\&-\beta_i(g_{kj}\eta H+g_{kh}\eta_j-2\eta_k\eta_j\eta_h)+(\alpha^2-\beta^2)(g_{kj}g_{ih}-g_{ki}g_{jh}-A_{jih}\eta_k)\\&-2\alpha\beta(g_{ki}\varphi_{jh}-g_{kj}\varphi_{ih}-B_{jih}\eta_k-B_{jik}\eta_h)].\end{aligned}\tag{3.8}$$

From (3.5) and (3.7), we have

$$\begin{aligned}n\{\xi(\beta)-(\alpha^2-\beta^2)\}(\alpha^2+\beta^2)R_{kjih}&=\big[(\alpha^2+\beta^2)[\tilde{\alpha}_k+(n-1)\{\beta_k-\xi(\beta)\eta_k\}]\\&+n\{\beta(\nabla_k\beta_l)+\alpha(\nabla_m\beta_l)\varphi_k{}^m\}\xi^l-2n(\alpha\beta\alpha_k-\beta^2\beta_k+\alpha^2\tilde{\alpha}_k-\alpha\beta\tilde{\beta}_k)\big]\{\alpha_j\varphi_{ih}\\&-\alpha_i\varphi_{jh}-\beta_j(g_{ih}-\eta_i\eta_h)+\beta_i(g_{jh}-\eta_j\eta_h)+(\alpha^2-\beta^2)A_{jih}-2\alpha\beta B_{jih}\}\\&+n\{\xi(\beta)-(\alpha^2-\beta^2)\}\big[\beta T_{kjih}+\alpha T_{mjih}\varphi_k{}^m+2(\alpha\beta\alpha_k-\beta^2\beta_k+\alpha^2\tilde{\alpha}_k-\alpha\beta\tilde{\beta}_k)A_{jih}\\&+2(\alpha^2\alpha_k+\alpha\beta\beta_k+\alpha\beta\tilde{\alpha}_k+\beta^2\tilde{\beta}_k)B_{jih}+(\alpha^2+\beta^2)[\{\alpha_j\varphi_{ih}-\alpha_i\varphi_{jh}-\beta_j(g_{ih}-\eta_i\eta_h)\\&+\beta_i(g_{jh}-\eta_j\eta_h)+(\alpha^2-\beta^2)A_{jih}-2\alpha\beta B_{jih}\}\eta_k+\alpha_jB_{hik}-\alpha_iB_{hjk}\\&+\beta_j(g_{ki}\eta_h+g_{kh}\eta_i-2\eta_k\eta_i\eta_h)-\beta_i(g_{hj}\eta_h+g_{kh}\eta_j-2\eta_k\eta_j\eta_h)\\&-(\alpha^2-\beta^2)(g_{kj}g_{ih}-g_{ki}g_{jh}-A_{jih}\eta_k)-2\alpha\beta(g_{ki}\varphi_{jh}-g_{kj}\varphi_{ih}-B_{jih}\eta_k-B_{jik}\eta_h)]\big].\end{aligned}\tag{3.9}$$

Thus we have

Theorem 3.3. *On a recurrent trans-Sasakian manifold with recurrent form* Π*, the Ricci tensor, the scalar curvature and the curvature tensor are given by* (3.6), (3.7) *and* (3.9)*, respectively.*

4. Properties of the recurrent form

In this section, we consider some properties of the recurrent form Π in a recurrent trans-Sasakian manifold.

Contraction of (3.5) by ξ^j and our assumption gives us

$$\{\xi(\beta)-(\alpha^2-\beta^2)\}\xi(\Pi)=2\{\xi^m\xi^l(\nabla_m\beta_l)-\xi(\alpha^2-\beta^2)\}.\tag{4.1}$$

Next, we have from (3.5) and (4.1)

$$\begin{aligned}n\{\xi(\beta)-(\alpha^2-\beta^2)\}(\beta\tilde{\Pi}_i-\alpha\Pi_i)&=(\alpha^2+\beta^2)\{\alpha_j-\xi(\alpha)\eta_j-(n-1)\tilde{\beta}_j\}\\&+n\{\beta(\nabla_m\beta_l)\varphi_j{}^m-\alpha(\nabla_j\beta_l)\}\xi^l+2n(\alpha2\alpha_j-\alpha\beta\beta_j-\alpha\beta\tilde{\alpha}_j+\beta^2\tilde{\beta}_j).\end{aligned}\tag{4.2}$$

From (3.5) and (4.2) we can easily obtain

$$\begin{aligned}n\{\xi(\beta)-(\alpha^2-\beta^2)\}\Pi_j&=-[\alpha\alpha_j+\beta\tilde{\alpha}_j+(n-1)(\beta\beta_j-\alpha\tilde{\beta}_j)\\&-\{\alpha\xi(\alpha)+(n-1)\xi(\beta)\}\eta_j]+n(\nabla_j\beta_l)\xi^l-2n(\alpha\alpha_j-\beta\beta_j).\end{aligned}\tag{4.3}$$

Thus we have

Theorem 4.1. *On a* $(2n+1)$*-dimensional recurrent trans-Sasakian manifold, the recurrent form* Π *satisfies* (4.3)*.*

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