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A NOTE ON PSEUDOSYMMETRIC LP-SASAKIAN MANIFOLDS IN THE CHAKI SENSE

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Abstract. This paper proves the existence of pseudosymmetric LP-Sasakian manifolds in the Chaki sense. We investigate the implications of η-parallel Ricci tensor and weakly locally ϕ-symmetric conditions on the geometry of this manifold. Furthermore, we analyze pseudo-Ricci-symmetric and pseudosymmetric LP-Sasakian manifolds (in the Chaki sense) under Einstein's field equations without cosmological constant.

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1. Introduction

Semi-Riemannian manifolds are essential in modern geometric physics, relativity and cosmology. If signature of n-dimensional semi-Riemannian manifolds is $(-,+,+,+,+,+,\ldots,+)$, then it is called *Lorentzian manifolds*, which play a vital role in these fields.

Sen and Chaki [20], in their 1967's publication, examined a particular curvature condition imposed on a conformal plane of the first class, thus obtaining the following expression for the covariant derivative of the Riemann curvature tensor ([7], [8]):

$$R^h_{ijk,l} = 2\lambda_l R^h_{ijk} + \lambda_i R^h_{ljk} + \lambda_j R^h_{ilk} + \lambda_k R^h_{ijl} + \lambda^h R_{lijk},$$

where R^h_{ijk} are the components of the curvature tensor R and R_{ijkl} is obtained by lowering the index h using the metric tensor g_{hl}, such that $R_{ijkl} = g_{hl} R^h_{ijk}$. Furthermore, λ_i is a non-zero covariant vector and the covariant derivative with respect to the metric tensor g_{ij} is denoted by ∇_k, such that $\nabla_k \lambda_i = \lambda_{i,k}$.

In 1987, Chaki [10] considered a non-flat Riemannian manifold (M^n, g), $n \geq 2$ whose $(1,3)$ type curvature tensor (R) is

$$(\nabla_X R)(Y,Z)W = 2A(X)R(Y,Z)W + A(Y)R(X,Z)W + A(Z)R(Y,X)W \tag{1.1}$$

$$+A(W)R(Y,Z)X+g(R(Y,Z)W,X)u,$$

where A is a non-zero 1-form, ∇ is the Levi-Civita connection, g is the metric tensor and u is the vector field, defined by $A(X)=g(X,u)\ \forall X\in(M^n,g)$. The manifold (M^n,g) becomes a symmetric manifold [9] if $A=0$.

In 2001, Arslan, Murathan, Ozgur, and Yıldız [1] extended the study of pseudosymmetric manifolds in the sense of Chaki to the context of contact metric manifolds.

Notable contributions to the study of pseudosymmetric manifolds have been advanced by the work of Sen and Chaki [20], Chaki [10], Arslan, Murathan, Ozgur and Yildiz [1] and many others.

A Lorentzian manifold ($n>3$), is called a *pseudo-Ricci-symmetric manifold* [5] if its Ricci tensor satisfies the following conditions:

$$(\nabla_X S)(Y,Z)=2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X). \tag{1.2}$$

Let M^n be a smooth n-dimensional manifold equipped with a pseudo-Riemannian metric g of Lorentzian signature $(-,+,...,+)$. M^n is a *Lorentzian Paracontact (LP) manifold* if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η such that:

$$g(X,\xi)=\eta(X);\quad \eta(\xi)=-1;\quad \phi(\xi)=0;\quad \eta(\phi)=0, \tag{1.3}$$

$$\phi^2X=X+\eta(X)\xi, \tag{1.4}$$

$$g(\phi X,\phi Y)=g(X,Y)+\eta(X)\eta(Y), \tag{1.5}$$

$$(\nabla_X\eta)(Y)=\Phi(X,Y)=\Phi(Y,X)=g(\phi X,Y), \tag{1.6}$$

$$\nabla_X\xi=\phi X,(\nabla_X\phi)(Y)=g(X,Y)\xi+\eta(Y)X+2\eta(X)\eta(Y)\xi, \tag{1.7}$$

for any X and $Y\in\chi(M)$. The following formulas are satisfied by LP-Sasakian manifolds ([15], [16], [6]):

$$R(X,Y)\xi=\eta(Y)X-\eta(X)Y, \tag{1.8}$$

$$R(\xi,X)Y=g(X,Y)\xi-\eta(Y)X, \tag{1.9}$$

$$R(\xi,Y)\xi=X+\eta(X)\xi, \tag{1.10}$$

$$S(X,\xi)=(n-1)\eta(X), \tag{1.11}$$

$$S(\phi X,\phi Y)=S(X,Y)+(n-1)\eta(X)\eta(Y). \tag{1.12}$$

Einstein's equation [17] of general relativity is the following ([3], [4]):

$$S(X,Y)-\frac{1}{2}rg(X,Y)+\lambda g(X,Y)=\kappa T(X,Y), \tag{1.13}$$

where r, $T(X,Y)$, λ and κ are the scalar curvature, the energy-momentum tensor, the cosmological constant and the gravitational constant, respectively. Then, Einstein's field equation without the cosmological constant is given by

$$S(X,Y)-\frac{1}{2}rg(X,Y)=\kappa T(X,Y). \tag{1.14}$$

Many authors have studied the semi-Riemannian manifolds (M^n, g), including Ozen [18], Guler and Altay [12], Petrov [19], Duggal and Sharma [11], Altin and ˙Unal [2], Majhi and Kar [14], O'Neill [17] and many others.

The present paper is organised as follows: In the Introduction section, we discuss very briefly the concepts of pseudosymmetry manifold in the Chaki sense, the LP-Sasakian manifold and Einstein's field equation. After the introduction, in Section 2, through an example, we show the existence of a pseudosymmetry in the Chaki sense on the LP-Sasakian manifolds. In Section 3, we explore the relationship between two 1-forms, one associated with pseudosymmetry manifolds and the other with LP-Sasakian manifolds. Furthermore, we analyze the impact of the η–parallel Ricci tensor condition and the weakly locally ϕ-symmetric condition on horizontal vector fields and their effect on the manifold's geometry. In the next section, we examine the structural features of pseudo-Ricci-symmetry manifolds, possessing Chaki pseudosymmetry, as defined within the context of LP-Sasakian manifolds. In the final Section, we determine the conditions under which pseudosymmetry manifolds in the Chaki sense on LP-Sasakian manifolds satisfy Einstein's field equations without a cosmological constant.

2. Example

Example 2.1. We will construct an example of a pseudosymmetric LP-Sasakian manifold. We will focus on a 3-dimensional manifold $M(\theta_1, \theta_2, \theta_3)$, where $(\theta_1, \theta_2, \theta_3 \in \Re)$ are the standard coordinates in $\Re$. The vector fields

$$E_1 = e^{\theta_3}\frac{\partial}{\partial\theta_2}; \quad E_2 = e^{\theta_3}(\frac{\partial}{\partial\theta_1} + \frac{\partial}{\partial\theta_2}); \quad E_3 = \frac{\partial}{\partial\theta_3},$$

are linearly independent at each point of M^n and

$$g(E_1, E_2) = g(E_2, E_3) = g(E_1, E_3) = 0,$$
$$g(E_1, E_1) = g(E_2, E_2) = 1, \quad g(E_3, E_3) = -1.$$

Let us consider $\eta(X) = g(X, E_3)$ and

$$\phi E_1 = -E_1, \quad \phi E_2 = -E_2, \quad \phi E_3 = 0.$$

Using the conditions stated above, we can demonstrate that $\eta(E_3) = -1, \phi^2 X = X + \eta(X)E_3$ and $g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y)$, for any $X, Y \in \chi(M)$. As a result, for $E_3 = \xi$ and

$$[E_1, E_2] = 0, \quad [E_1, E_3] = -E_1, \quad [E_2, E_3] = -E_2.$$

Taking Koszul formula for the Lorentzian metric g, we can easily calculate that

$$\nabla_{E_1}E_3 = -E_1, \quad \nabla_{E_1}E_2 = 0, \quad \nabla_{E_1}E_1 = E_3,$$
$$\nabla_{E_2}E_3 = -E_2, \quad \nabla_{E_2}E_2 = E_3, \quad \nabla_{E_2}E_1 = 0,$$
$$\nabla_{E_3}E_3 = 0, \quad \nabla_{E_3}E_2 = 0, \quad \nabla_{E_3}E_1 = 0.$$

Based on the preceding relations, it follows that the semi-Riemannian manifold M is an LP-Sasakian manifold. We can calculate that

$$R(E_1, E_2)E_3 = 0, \; R(E_2, E_3)E_3 = -E_2, \; R(E_1, E_3)E_3 = -E_1,$$

$$R(E_1, E_2)E_2 = -E_1, \; R(E_2, E_3)E_2 = E_3, \; R(E_1, E_3)E_2 = 0,$$

$$R(E_1, E_2)E_1 = -E_2, \; R(E_2, E_3)E_1 = 0, \; R(E_1, E_3)E_1 = E_3.$$

With the help of all the above relations and (1.1), we prove that

$$(\nabla_{E_2}R)(E_2,E_3)E_3=-E_3,\ \ (\nabla_{E_1}R)(E_1,E_3)E_3=-E_3,$$

$$(\nabla_{E_1}R)(E_2,E_3)E_2=-E_1,\ \ (\nabla_{E_1}R)(E_1,E_2)E_2=-E_3,$$

$$(\nabla_{E_2}R)(E_1,E_2)E_1=-E_3,\ \ (\nabla_{E_2}R)(E_2,E_3)E_2=-E_2,$$

$$(\nabla_{E_1}R)(E_1,E_3)E_1=-E_1,\ \ (\nabla_{E_2}R)(E_1,E_3)E_1=-E_2.$$

We can conclude from the above arguments that the LP-Sasakian manifold with structure $M(\phi,\xi,\eta,g)$ is 3-dimensional pseudosymmetric manifold according to Chaki's definition.

3. Pseudosymmetric manifold of Chaki type

Theorem 3.1. *On an n-dimensional LP-Sasakian manifold, where $n\neq 2$, the two 1-forms associated with the pseudosymmetric condition (one arising from the pseudosymmetry of the manifold itself and the other from the LP-Sasakian structure) are scalar multiples of each other.*

Proof. If M^n is a Chaki-type pseudosymmetric manifold and using equation (1.1), then we have

$$(\nabla_X S)(Y,Z)=2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X)+A(R(X,Y)Z)+A(R(X,Z)Y). \tag{3.1}$$

Putting $Z=\xi$ in Equation (3.1), we obtain

$$(\nabla_X S)(Y,\xi)=2A(X)S(Y,\xi)+A(Y)S(X,\xi)+A(\xi)S(Y,X)+A(R(X,Y)\xi)+A(R(X,\xi)Y). \tag{3.2}$$

Combining the equations (1.8), (1.9), (1.11) and (3.2), we get

$$(\nabla_X S)(Y,\xi)=2nA(X)\eta(Y)+(n-2)A(Y)\eta(X)+A(\xi)S(Y,X)-A(\xi)g(X,Y). \tag{3.3}$$

Again putting $X=\xi$ in equation (3.3), we write

$$(\nabla_\xi S)(Y,\xi)=(3n-2)A(\xi)\eta(Y)-(n-2)A(Y). \tag{3.4}$$

With the help of equations (1.6), (1.7) and $(\nabla_\xi S)(Y,\xi)=\nabla_\xi S(Y,\xi)-S(\nabla_\xi Y,\xi)-S(Y,\nabla_\xi\xi)$, we have

$$(\nabla_\xi S)(Y,\xi)=0. \tag{3.5}$$

From the equations (3.4) and (3.5), we conclude that

$$u=\frac{(3n-2)A(\xi)}{n-2}\xi. \tag{3.6}$$

The theorem has been proved. □

Definition 3.2. [13] An LP-Sasakian manifold is called an *η-parallel Ricci tensor* if the Ricci tensor satisfies

$$(\nabla_X S)(\phi Y,\phi Z)=0, \tag{3.7}$$

for any X,Y,Z.

Theorem 3.3. *Let (M^n,g) be an LP-Sasakian manifold that is pseudosymmetric in the Chaki sense. If the Ricci tensor of (M^n,g) is η-parallel and $A(\xi)\neq 0$, then the scalar curvature $r=2(1-n)$.*

Proof. We can write that

$$(\nabla_X S)(\phi Y,\phi Z)=\nabla_X S(\phi Y,\phi Z)-S(\nabla_X\phi Y,\phi Z)-S(\phi Y,\nabla_X\phi Z). \tag{3.8}$$

Combining the equations (1.3), (1.7), (1.11), (1.12) and (3.8), it follows that

$$\begin{aligned}(\nabla_X S)(\phi Y,\phi Z)&=(\nabla_X S)(Y,Z)-S(X,\phi Z)\eta(Y)-S(X,\phi Y)\eta(Z)+(n-1)(\nabla_X\eta)(Y)\eta(Z)\\&\quad+(n-1)(\nabla_X\eta)(Z)\eta(Y).\end{aligned} \tag{3.9}$$

With the help of equations (1.6) and (3.9), we get

$$\begin{aligned}(\nabla_X S)(\phi Y,\phi Z)&=(\nabla_X S)(Y,Z)-S(X,\phi Z)\eta(Y)-S(X,\phi Y)\eta(Z)+(n-1)g(\phi X,Y)\eta(Z)\\&\quad+(n-1)g(\phi X,Z)\eta(Y).\end{aligned} \tag{3.10}$$

Adding the equations (3.7) and (3.10), we can write that

$$\begin{aligned}(\nabla_X S)_{ric}(Y,Z)&=S(X,\phi Z)\eta(Y)+S(X,\phi Y)\eta(Z)-(n-1)g(\phi X,Y)\eta(Z)\\&\quad-(n-1)g(\phi X,Z)\eta(Y).\end{aligned} \tag{3.11}$$

From (3.1) and (3.11), it implies that

$$\begin{aligned}&2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X)+A(R(X,Y)Z)+A(R(X,Z)Y)\\&=S(X,\phi Z)\eta(Y)+S(X,\phi Y)\eta(Z)-(n-1)g(\phi X,Y)\eta(Z)-(n-1)g(\phi X,Z)\eta(Y).\end{aligned} \tag{3.12}$$

Putting $X=\xi$ in equation (3.12) and using (1.3), (1.9) and (1.11), we get

$$2A(\xi)S(Y,Z)+(n-2)[A(Y)\eta(Z)+A(Z)\eta(Y)]+2A(\xi)g(Y,Z)=0. \tag{3.13}$$

Using (1.3) in (3.13), we obtain

$$2A(\xi)[r+2n-2]=0.$$

That means if $A(\xi)\neq 0$, then $r=2(1-n)$. The proof of the theorem is completed. □

Definition 3.4. [21] The LP-Sasakian manifolds (M^n,g) is said to be *weakly locally ϕ-symmetric* if

$$\phi^2((\nabla_X R)(Y,Z)W)=0, \tag{3.14}$$

holds for any vector field X,Y,Z,W orthogonal to ξ, that is, for any horizontal vector fields X,Y,Z,W.

Theorem 3.5. *If the Chaki-pseudosymmetric curvature tensor of an LP-Sasakian manifold (M^n,g) is weakly locally ϕ-symmetric for all horizontal vector fields, then the manifold is an Einstein manifold.*

Proof. With the help of the equations (1.1), (1.3), (1.4) and (3.14), we can prove that

$$\begin{aligned}&2A(X)R(Y,Z)W+A(Y)R(X,Z)W+A(Z)R(Y,X)W+A(W)R(Y,Z)X\\&+g(R(Y,Z)W,X)u+2A(X)\eta(R(Y,Z)W)\xi+A(Y)\eta(R(X,Z)W)\xi\\&+A(Z)\eta(R(Y,X)W)\xi+A(W)\eta(R(Y,Z)X)\xi\\&+g(R(Y,Z)W,X)\eta(u)\xi=0.\end{aligned} \tag{3.15}$$

If X,Y and Z are orthogonal to ξ, then $\eta(X)=\eta(Y)=\eta(Z)=0$, putting $W=\xi$ and using (1.8) in (3.15), we get

$$R(Y,Z)X+\eta(R(Y,Z)X)\xi=0. \tag{3.16}$$

We can rewrite the equation (3.16) to another form,

$$S(X,Z)+g((R(\xi,Z)X),\xi)=0. \tag{3.17}$$

Using the equations (1.3), (1.9) and (3.17), we can conclude that

$$S(X,Z)=g(X,Z).$$

That means the manifold is an Einstein manifold. Finally, the theorem has been proven. □

4. Pseudo-Ricci-symmetric manifold of Chaki type

Theorem 4.1. *When an LP-Sasakian manifold (M^n,g) is Chaki-pseudo-Ricci symmetric, then it satisfies the relation $A(\xi)S(Y,X)-S(\phi X,Y)=(n-1)[g(\phi X,Y)-2A(X)\eta(Y)-A(Y)\eta(X)]$.*

Proof. We take $Z=\xi$ in (1.2) and using (1.11), it implies that

$$(\nabla_X S)(Y,\xi)=2(n-1)A(X)\eta(Y)+(n-1)A(Y)\eta(X)+A(\xi)S(Y,X). \tag{4.1}$$

Put $Z=\xi$ in the relationship $(\nabla_X S)(Y,Z)=\nabla_X S(Y,Z)-S(\nabla_X Y,Z)-S(Y,\nabla_X Z)$ and using (1.7), it follows that

$$(\nabla_X S)(Y,\xi)=(n-1)(\nabla_X\eta)(Y)-S(\phi X,Y). \tag{4.2}$$

Combining the equations (1.6) and (4.2), we get

$$(\nabla_X S)(Y,\xi)=(n-1)g(\phi X,Y)-S(\phi X,Y). \tag{4.3}$$

From the equations (4.1) and (4.3), we have

$$A(\xi)S(Y,X)-S(\phi X,Y)=(n-1)[g(\phi X,Y)-2A(X)\eta(Y)-A(Y)\eta(X)].$$

The proof of the theorem is complete. □

Definition 4.2. An LP-Sasakian manifold is said to be *Ricci parallel manifold* if the Ricci tensor satisfies the following:

$$(\nabla_X S)(Y,Z)=0, \tag{4.4}$$

for any X,Y,Z.

Theorem 4.3. *Let (M^n,g) be an LP-Sasakian manifold that is Chaki-pseudo-Ricci symmetric. If the Ricci tensor is parallel and the dimension n is 1, then the manifold is Ricci flat.*

Proof. We obtain from the equations (1.2) and (4.4),

$$2A(X)S(Y,Z)+A(Y)S(X,Z)+A(Z)S(Y,X)=0. \tag{4.5}$$

Putting $X=\xi$ in (4.5) and using (1.11), we can write that

$$2A(\xi)S(Y,Z)+(n-1)[A(Y)\eta(Z)+A(Z)\eta(Y)]=0. \tag{4.6}$$

If $n=1$, $S(Y,Z)=0$. That means the Ricci tensor vanishes, which means that the manifold is Ricci flat. □

5. Energy momentum tensor without cosmological constant

Theorem 5.1. *Let (M^n, g) be an LP-Sasakian manifold. If (M^n, g) is Chaki-pseudosymmetric and satisfies Einstein's field equation without cosmological constant, then (M^n, g) is Ricci-flat if and only if its energy-momentum tensor is divergence-free, provided the vector fields of the manifolds are horizontal.*

Proof. We can change the form of the equation (1.14),

$$(\nabla_W S)(X,Y) - \frac{1}{2}(\nabla_W r)g(X,Y) = \kappa(\nabla_W T)(X,Y). \tag{5.1}$$

Considering the equations (1.2) and (5.1), it follows that

$$\kappa(\nabla_W T)(X,Y) = 2A(W)S(X,Y) + A(X)S(W,Y) + A(Y)S(X,W) - \frac{1}{2}(\nabla_W r)g(X,Y). \tag{5.2}$$

We take $Y = \xi$ in (5.2) and using (1.3) and (1.11), it implies that

$$\kappa(\nabla_W T)(X,\xi) = (n-1)[2A(W)\eta(X) + A(X)\eta(W)] + A(\xi)S(X,W) - \frac{1}{2}(\nabla_W r)\eta(X). \tag{5.3}$$

If X and W are horizontal vector fields, $\eta(X) = \eta(W) = 0$. Then we can write the equation (5.3),

$$\kappa(\nabla_W T)(X,\xi) = A(\xi)S(X,W).$$

That means the manifold is Ricci flat if and only if the energy momentum tensor is divergence-free. The proof of the theorem is completed. ☐

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