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MATHEMATICAL MODELLING OF WAVES IN A HOMOGENEOUS ISOTROPIC ROTATING CYLINDRICAL PANEL

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Abstract

The three dimensional wave propagation in a homogeneous isotropic rotating cylindrical panel is investigated in the context of the linear theory of elasticity. Three displacement potential functions are introduced to uncouple the equations of motion. The frequency equations are obtained using the traction free boundary conditions. Modified Bessel functions with complex arguments are directly used to analyze the frequency equations and are studied numerically for the material copper. Since the speed of the disturbed waves depend upon rotation rate, this type of study is important in the design of high speed steam , gas turbine and rotation rate sensors. The computed non-dimensional frequencies are plotted in the form of dispersion curves with the support of MATLAB.

Keywords: Isotropic cylindrical panel, Rotation, modified Bessel function.

Mathematics Subject Classification: 74H45,74J20, 74K25.

1. Introduction

The effect of rotation on cylindrical panels has its applications in the diverse engineering field like civil, architecture, aeronautical and marine engineering. At the present time applied mathematicians are exhibiting considerable interest in dynamical methods of elasticity, since the usual quasi static approach ignores certain very important features of the problems under consideration. That approach is based on the assumption that the inertia terms may be omitted from the equations of motion. This assumption holds good only when the variations in stresses and displacements, but there arise number of problems in engineering and technology, when this assumption may not hold good and the inertia terms in the equations of motion may have lead to cases of considerable mathematical complications. In the field of nondestructive evaluation, laser-generated waves have attracted great attention owing to their potential application to noncontact and nondestructive evaluation of sheet materials. This study may be used in applications involving nondestructive testing (NDT), qualitative nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

The theory of elastic vibrations and waves is well established [1]. An excellent collection of works on vibration of shells were published by Leissa [2]. Mirsky [3] analyzed the wave propagation in transversely isotropic circular cylinder of infinite length and presented the numerical results. Gazis [4] has studied the most general form of harmonic waves in a hollow cylinder of infinite length. Sinha et. al. [5] have discussed the axisymmetric wave propagation in circular cylindrical shell immersed in fluid in two parts. In Part I, the theoretical analysis of the propagating modes are discussed and in Part II, the axisymmetric modes excluding torsional modes are obtained theoretically and experimentally and are compared. Vibration of functionally graded multilayered orthotropic cylindrical panel under thermo mechanical load was analyzed by Wang et.al [6]. Three dimensional vibration of a homogenous transversely isotropic thermo elastic cylindrical panel was investigated by Sharma [7]. Free vibration of transversely isotropic piezoelectric circular cylindrical panels was studied by Ding et.al [8]. An iterative approach predicts the frequency of isotropic cylindrical shell and panel was studied by Soldatos and Hadhgeorgian [9]. Chen [10] developed a mathematical modeling for free vibration of orthotropic cylindrical shell by using Rayleigh–Ritz method. Free vibration of composite cylindrical panels with random material properties was developed by Sing et.al [11], in this work the effect of variations in the mechanical properties of laminated composite cylindrical panels on its natural frequency has been obtained by modeling these as random variables. Zhang [12] employed a wave propagation method to analysis the frequency of cylindrical panels. Lam and Loy [13] investigated the vibration of thin cylindrical panels of simply supported boundary conditions with Flugge's theory and also studied the vibration of rotating cylindrical panel. Ponnusamy and Selvamani [14,15] obtained the frequency equation for free vibration of homogeneous isotropic and generalized thermo elastic cylindrical panel and presented the numerical result for Zinc. The theory of elastic material with rotation is plays a vital role in civil, architecture, aeronautical and marine engineering. Body wave propagation in rotating thermo elastic media was investigated by Sharma and Grover [16].The effect of rotation, magneto field, thermal relaxation time and pressure on the wave propagation in a generalized visco elastic medium under the influence of time harmonic source is discussed by Abd-Alla and Bayones [17].The propagation of waves in conducting piezoelectric solid is studied for the case when the entire medium rotates with a uniform angular velocity by Wauer [18]. Roychoudhuri and Mukhopadhyay studied the effect of rotation and relaxation times on plane waves in generalized thermo visco elasticity[19].Gamer [20] discussed the elastic-plastic deformation of the rotating solid disk. Lam [21] studied the frequency characteristics of a thin rotating cylindrical shell using general differential quadrature method.

In this paper, the three dimensional wave propagation in a homogeneous isotropic rotating cylindrical panel is discussed using the linear three-dimensional theory of elasticity. The frequency equations are obtained using the traction free boundary conditions. A modified Bessel functions with complex argument is directly used to analyze the frequency by fixing the circumferential wave number and are studied numerically for the material copper. The computed non-dimensional frequencies are plotted in the form of dispersion curves.

2. The Governing equations

Consider a cylindrical panel as shown in Fig.1 of length L having inner and outer radius a and b with thickness h. The angle subtended by the cylindrical panel, which is known as center angle, is denoted by α . The deformation of the cylindrical panel in the direction r, θ, z are defined by u, v and w . The cylindrical panel is assumed to be homogenous, isotropic and linearly elastic with a rotational speed Ω, Young's modulus E, poisson ratio ν and density ρ in an undisturbed state

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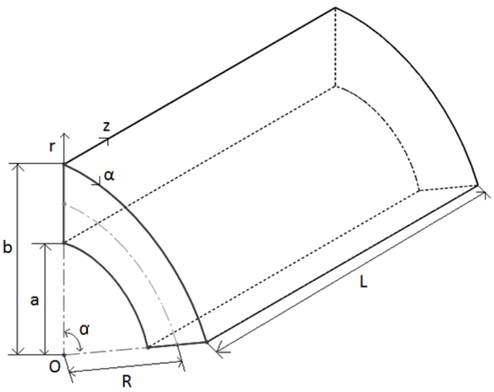

Fig.1 Cylindrical panel

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In cylindrical coordinate the three dimensional stress equation of motion, strain displacement relation in the absence of body force for a linearly elastic medium rotating about the z-axis from[16]

$$\sigma_{rr,r}+r^{-1}\sigma_{r\theta,\theta}+\sigma_{rz,z}+r^{-1}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)+\rho\Omega^2u=\rho u_{,tt}$$

$$\sigma_{r\theta,r}+r^{-1}\sigma_{\theta\theta,\theta}+\sigma_{,rzz}+\sigma_{\theta z,z}+2r^{-1}\sigma_{r\theta}=\rho v_{,tt} \quad \textbf{(1)}$$

$$\sigma_{rz,r}+r^{-1}\sigma_{\theta z,\theta}+\sigma_{zz,z}+r^{-1}\sigma_{r\theta}=\rho w_{,tt}$$

where ρ is the mass density, Ω is the uniform angular velocity.

$$\sigma_{rr}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{rr}$$

$$\sigma_{\theta\theta}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{\theta\theta} \quad \textbf{(2)}$$

$$\sigma_{zz}=\lambda(e_{rr}+e_{\theta\theta}+e_{zz})+2\mu e_{zz}$$

where e_{ij} are the strain components, t is the time, λ and μ are Lame' constants. The strain e_{ij} are related to the displacements are given by

$$\sigma_{r\theta}=\mu\gamma_{r\theta} \quad \sigma_{rz}=\mu\gamma_{rz} \quad \sigma_{\theta z}=\mu\gamma_{\theta z} \quad e_{rr}=\frac{\partial u}{\partial r} \quad e_{\theta\theta}=\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta} \quad \textbf{(3)}$$

$$e_{zz}=\frac{\partial w}{\partial z} \quad \gamma_{r\theta}=\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta} \quad \gamma_{rz}=\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z} \quad \gamma_{z\theta}=\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial w}{\partial\theta} \quad \textbf{(4)}$$

where u,v,w are displacements along radial, circumferential and axial directions respectively. $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz}$ are the normal stress components and $\sigma_{r\theta},\sigma_{\theta z},\sigma_{zr}$ are the shear stress components , $e_{rr},e_{\theta\theta},e_{zz}$ are normal strain components and $e_{r\theta},e_{\theta z},e_{zr}$ are shear strain components.

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Substituting the equations(2),(3) and (4) in equation(1),gives the following three displacement equations of motion :

$$
\begin{aligned}
&(\lambda+2\mu)\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+\mu r^{-2}u_{,\theta\theta}+\mu u_{,zz}+r^{-1}(\lambda+\mu)v_{,r\theta}-r^{-2}(\lambda+3\mu)v_{,\theta}\\
&+(\lambda+\mu)w_{,rz}+\rho\Omega^2 u=\rho u_{,tt}\\
&\mu\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+r^{-2}(\lambda+2\mu)v_{,\theta\theta}+\mu v_{,zz}+r^{-2}(\lambda+3\mu)u_{,\theta}+r^{-1}(\lambda+\mu)u_{,r\theta}\\
&+r^{-1}(\lambda+\mu)w_{,\theta z}=\rho v_{,tt}\\
&(\lambda+2\mu)w_{,zz}+\mu\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+(\lambda+\mu)u_{,rz}+r^{-1}(\lambda+\mu)v_{,\theta z}\\
&+r^{-1}(\lambda+\mu)u_{,z}=\rho w_{,tt}\,.
\end{aligned}
\tag{5}
$$

To solve equation (5), we take [7]

$$u=\frac{1}{r}\psi,_{\theta}-\phi,_{r}\quad v=-\frac{1}{r}\phi,_{\theta}-\psi,_{\varpi}\quad w=-\chi,_{z}$$

Using Eqs (5) in Eqs (1), we find that ϕ,χ,T satisfies the equations.

$$((\lambda+2\mu)\nabla^2{}_1+\mu\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2}+\rho\Omega^2)\phi-(\lambda+\mu)\frac{\partial^2\chi}{\partial z^2}^{2}=0 \tag{6a}$$

$$(\mu\nabla_1{}^2+(\lambda+2\mu)\frac{\partial^2}{\partial z^2}-\rho\frac{\partial^2}{\partial t^2})\chi-(\lambda+\mu)\nabla_1{}^2\phi=0 \tag{6b}$$

$$(\nabla_1{}^2+\frac{\partial^2}{\partial z^2}-\frac{\rho}{\mu}\frac{\partial^2}{\partial t^2}-\rho\Omega^2)\psi=0\ . \tag{6c}$$

where $\quad \nabla_1{}^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$

Equation (6c) in ψ gives a purely transverse wave. This wave is polarized in planes perpendicular to the z-axis. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$.

3. Solution to the problem

The equation (6) is coupled partial differential equations of the three displacement components. To uncouple equations (6), we can write three displacement functions which satisfies the simply supported boundary conditions followed by Sharma [7]

$$\psi(r,\theta,z,t)=\overline{\psi}(r)\sin(m\pi z)\cos(n\pi\theta/\alpha)e^{i\omega t}$$
$$\phi(r,\theta,z,t)=\overline{\phi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t} \tag{7}$$
$$\chi(r,\theta,z,t)=\overline{\chi}(r)\sin(m\pi z)\sin(n\pi\theta/\alpha)e^{i\omega t}$$

where m is the circumferential mode and n is the axial mode, ω is the angular frequency of the cylindrical panel motion. By introducing the dimensionless quantities

$$r^{'}=\frac{r}{R} \qquad z^{'}=\frac{z}{L} \qquad \delta=\frac{n\pi}{\alpha} \qquad t_L=\frac{m\pi R}{L} \qquad \overline{\lambda}=\frac{\lambda}{\mu} \qquad \in_1=\frac{1}{2+\overline{\lambda}}$$

$$C_1^{\,2}=\frac{\lambda+2\mu}{\rho} \qquad \varpi^2=\frac{\omega^2R^2}{C^2_1} \qquad \Gamma=\frac{\rho\Omega^2R^2}{2+\overline{\lambda}}\ . \tag{8}$$

After substituting equation (8) in (7) ,we obtain the following system of equations :

$$(\nabla_2^2+k_1^{\,2})\overline{\psi}=0 \tag{9a}$$

$$(\nabla_2^2+g_1)\overline{\phi}+g_2\overline{\chi}=0 \tag{9b}$$

$$(\nabla_2^2+g_3)\overline{\chi}+(1+\overline{\lambda})\nabla_2^2\overline{\phi}=0 \tag{9c}$$

where

$$\nabla_2^2=\frac{\partial^2}{\partial r^2}\frac{1}{r}\frac{\partial}{\partial r}-\frac{\delta^2}{r^2}\ ,$$

$$g_1=(2+\overline{\lambda})(t_L^2-\varpi^2+\Gamma) \qquad g_2=\in_1(1+\overline{\lambda})t_L^{\,2} \qquad g_3=(\varpi^2-\in_1 t_L^{\,2})$$

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C_1 wave velocity of the cylindrical panel. A non-trivial solution of the algebraic equations systems (9) exist only when the determinant of equations (9) is equal to zero.

$$\begin{vmatrix} \left(\nabla_2^2 + g_3\right) & -g_2 \\ \left(1+\bar{\lambda}\right)\nabla_2^2 & \left(\nabla_2^2 - g_1\right) \end{vmatrix} \left(\bar{\phi}, \bar{\chi}\right) = 0 . \qquad \textbf{(10)}$$

The Eq. (10), on simplification reduces to the following differential equation. In addition, α_1 and α_2 ($\mathrm{Re}\left(\alpha_1 \geq 0\right)$ and $\mathrm{Re}\left(\alpha_2 \geq 0\right)$) are the two roots of the following equation

$$\left(\nabla_2^4 + B\nabla_2^2 + C\right)\bar{\phi} = 0 \qquad \textbf{(11)}$$

where

$$B = -g_1 + g_2(1+\bar{\lambda}) + g_3 \qquad C = -g_1 g_3 \; .$$

The solution of equation (11) is

$$\bar{\phi}(r) = \sum_{i=1}^{2} \left[A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r) \right]$$

$$\bar{\chi}(r) = \sum_{i=1}^{2} d_i \left[A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r) \right] . \qquad \textbf{(12)}$$

Here, $\left(\alpha_i r\right)^2$ are the non-zero roots of the algebraic equation

$$\left(\alpha_i r\right)^4 + B\left(\alpha_i r\right)^2 - C = 0 . \qquad \textbf{(13)}$$

The arbitrary constant d_i is obtained from

$$d_i = \frac{\left(1+\overline{\lambda}\right)\left(\alpha_i r\right)^2}{\left(\alpha_i r\right)^2 + g_3} \ . \tag{14}$$

Eq. (9a) is a Bessel equation with its possible solutions is

$$\overline{\psi} = \begin{cases} A_3 J_\delta(k_1 r) + B_3 Y_\delta(k_1 r), & k_1^2 > 0 \\ A_3 r^\delta + B_3 r^{-\delta}, & k_1^2 = 0 \\ A_3 I_\delta(k_1' r) + B_3 K_\delta(k_1' r), & k_1^2 < 0 \end{cases} \tag{15}$$

where $k_1^{'2} = -k_1^2$ and J_δ and Y_δ are Bessel functions of the first and second kinds respectively while, I_δ and k_δ are modified Bessel functions of first and second kinds respectively. A_3 and B_3 are two arbitrary constants. Generally $k_1^2 \neq 0$,so that the situation $k_1^2 \neq 0$ is will not be discussed in the following . For convenience, we consider the case of $k_1^2 > 0$,and the derivation for the case of $k_1^2 < 0$ is similar.

The solution of equation (9a) is

$$\overline{\psi}(r) = A_3 J_\delta(k_1 r) + B_3 Y_\delta(k_1 r) \tag{16}$$

where $k_1{}^2 = (2+\overline{\lambda})\Omega^2 - \left(t_L{}^2 + \Gamma\right)$.

3.1 Elastokinetic

In the present analysis if we take the coupling parameter for magnetic and thermal field $\Gamma = 0$, then the equations from (10) - (12) will reduces to the classical case in elastokinetic

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$$\begin{vmatrix} \left(\nabla_2^2 + g_3\right) & -g_2 \\ \left(1+\bar{\lambda}\right)\nabla_2^2 & \left(\nabla_2^2 - g_1\right) \end{vmatrix} \left(\bar{\phi}, \bar{\chi}\right) = 0 \tag{17}$$

$$\left(\nabla_2^4 + A_3\nabla_2^2 + B_3\right)\bar{G} = 0 \tag{18}$$

$$A_3 = g_1 + \left(1+\bar{\lambda}\right) g_2 + g_3$$

$$B_3 = g_1 g_3$$

$$\bar{\phi}(r) = \sum_{i=1}^{2} \left[A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r) \right]$$

$$\bar{\chi}(r) = \sum_{i=1}^{2} d_i \left[A_i J_\delta(\alpha_i r) + B_i Y_\delta(\alpha_i r) \right] \tag{19}$$

$$d_i = \frac{\left(1+\bar{\lambda}\right)\left(\alpha_i r\right)^2}{\left(\alpha_i r\right)^2 + g_3} \tag{20}$$

Equations (17)-(20) constitute the solution for the homogenous isotropic cylindrical panel with traction free boundary conditions. It is noticed that equation (18) is similar to one as obtained and discussed by Chen et al[21] in case of elastokinetics.

4. Boundary condition and frequency equation

In this section we shall derive the secular equation for the three dimensional vibrations cylindrical panel subjected to traction free boundary conditions at the upper and lower surfaces at

$$r = a, b$$

$$u=\left(-\bar{\phi}^{'}-\frac{\delta\bar{\psi}^{'}}{r}\right)\sin(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$v=\left(-\bar{\psi}^{'}-\frac{\delta\bar{\phi}^{'}}{r}\right)\sin(m\pi z)\cos(\delta\theta)e^{i\omega t}$$

$$w=\bar{\chi}t_L\cos(m\pi z)\sin(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{rr}=\left[\begin{array}{l}\left(2+\bar{\lambda}\right)\delta\left(\frac{\bar{\psi}^{'}}{r}-\frac{\bar{\psi}}{r^2}\right)+\left(2+\bar{\lambda}\right)\left(\frac{1}{r}\bar{\varphi}^{'}+(\alpha_i^2-\frac{\delta^2}{r^2}\bar{\varphi})\right)\\+\bar{\lambda}\left(\frac{\delta}{r^2}\bar{\psi}-\frac{1}{r}\bar{\phi}^{'}-\frac{\delta^2}{r^2}\bar{\varphi}-\frac{\delta}{r}\bar{\psi}^{'}-t_L^2\bar{\chi}\right)\end{array}\right]\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{r\theta}=2\left(\frac{1}{r}\bar{\psi}+(\alpha_i^2-\frac{\delta^2}{r^2})\bar{\psi}-\frac{2\delta}{r}\bar{\varphi}^{'}+\frac{2\delta}{r^2}\bar{\varphi}+\frac{\bar{\psi}^{'}}{r}-\frac{\delta^2}{r^2}\bar{\psi}\right)\sin(m\pi)z\cos(\delta\theta)e^{i\omega t}$$

$$\bar{\sigma}_{rz}=2t_L\left(-\bar{\varphi}^{'}-\frac{\delta}{r}\bar{\psi}+\bar{\chi}^{'}\right)\cos(m\pi)z\sin(\delta\theta)e^{i\omega t}$$

Where prime denotes the differentiation with respect to r $\bar{u}_i=u_i/R,(i=r,\theta,z)$ are three non– dimensional displacements and $\bar{\sigma}rr=\sigma_{rr}/\mu,\bar{\sigma}r\theta=\sigma_{r\theta}/\mu,\bar{\sigma}rz=\sigma_{rz}/\mu$ are three non-dimensional stresses. In this case both convex and concave surface of the panel are traction free

$$\sigma_{rr}=\sigma_{r\theta}=\sigma_{rz}=0 \qquad (r=a,b) \tag{21}$$

Using the result obtained in the equations (1)-(3) in (13) we can get the frequency equation of free vibration as follows

$$\left|E_{ij}\right|=0 \qquad i,j=1,2,...6 \tag{22}$$

$$E_{11}=(2+\bar{\lambda})\left((\delta J_\delta(\alpha_1t_1)/t_1^{\,2}-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1))-((\alpha_1t_1)^2R^2-\delta^2)J_\delta(\alpha_1t_1)/t_1^{\,2}\right)$$

$$+\bar{\lambda}\left(\delta(\delta-1)J_\delta(\alpha_1t_1)/t_1^{\,2}-\frac{\alpha_1}{t_1}J_{\delta+1}(\alpha_1t_1)\right)+\bar{\lambda}d_1t_L^{\,2}J_\delta(\alpha_1t_1)$$

$$E_{13}=(2+\overline{\lambda})\left((\delta J_{\delta}(\alpha_2 t_1)/t_1^{\,2}-\frac{\alpha_2}{t_2}J_{\delta+1}(\alpha_2 t_1))-((\alpha_2 t_1)^2 R^2-\delta^2)J_{\delta}(\alpha_2 t_1)/t_1^{\,2}\right)$$
$$+\overline{\lambda}\left(\delta(\delta-1)J_{\delta}(\alpha_2 t_1)/t_1^{\,2}-\frac{\alpha_2}{t_1}J_{\delta+1}(\alpha_2 t_1)\right)+\overline{\lambda}d_2 t_L^{\,2}J_{\delta}(\alpha_2 t_1)$$

$$E_{15}=(2+\overline{\lambda})\left((\frac{k_1\delta}{t_1}J_{\delta+1}(k_1 t_1)-\delta(\delta-1)J_{\delta}(k_1 t_1)/t_1^{\,2}\right)$$
$$+\overline{\lambda}\left(\delta(\delta-1)J_{\delta}(k_1 t_1)/t_1^{\,2}-\frac{k_1\delta}{t_1}J_{\delta+1}(k_1 t_1)\right)$$

$$E_{21}=2\delta\left((\alpha_1/t_1)J_{\delta+1}(\alpha_1 t_1)-\delta(\delta-1)J_{\delta}(\alpha_1 t_1)\right)$$

$$E_{23}=2\delta\left((\alpha_2/t_1)J_{\delta+1}(\alpha_2 t_1)-\delta(\delta-1)J_{\delta}(\alpha_2 t_1)\right)$$

$$E_{25}=(k_1 t_1)^2 R^2 J_{\delta}(k_1 t_1)-2\delta(\delta-1)J_{\delta}(k_1 t_1)/t_1^{\,2}+k_1/t_1 J_{\delta+1}(k_1 t_1)$$

$$E_{31}=-t_L(1+d_1)\left(\delta/t_1 J_{\delta}(\alpha_1 t_1)-\alpha_1 J_{\delta+1}(\alpha_1 t_1)\right)$$

$$E_{33}=-t_L(1+d_2)\left(\delta/t_1 J_{\delta}(\alpha_2 t_1)-\alpha_2 J_{\delta+1}(\alpha_2 t_1)\right)$$

$$E_{35}=-t_L(\delta/t_1)J_{\delta}(k_1 t_1)$$

in which $t_1=a/R=1-t^*/2$, $t_2=b/R=1+t^*/2$ and $t^*=b-a/R$ is the thickness - to-mean radius ratio of the panel. Obviously $E_{ij}\left(j=2,4,6\right)$ can obtained by just replacing modified Bessel function of the first kind in $E_{ij}\left(i=1,3,5\right)$ with the ones of the second kind, respectively, while $E_{ij}\left(i=4,5,6\right)$ can be obtained by just replacing t_1 in $E_{ij}\left(i=1,2,3\right)$ with t_2 .

5. Numerical results and discussion

The frequency equation (22) is numerically solved for Zinc material. For the purpose of numerical computation we consider the closed circular cylindrical shell with the center angle $\alpha = 2\pi$ and the integer n must be even since the shell vibrates in circumferential full wave. The frequency equation for a closed cylindrical shell can be obtained by setting $\delta = l\left(l = 1, 2, 3.....\right)$ where l is the circumferential wave number in equations(22). The material properties of a copper is

$\rho = 8.96\times10^{3} kgm^{-3}$,

$\mu = 4.20\times10^{11} Kgms^{-2}$, $\lambda = 8.20\times10^{11} Kgms^{-2}$ and

$\nu = 0.3$, $E = 2.139\times10^{11} Nm^{-2}$

The roots of the algebraic equation (22) was calculated using a combination of Birge-Vita method and Newton-Raphson method. In the present case simple Birge-Vita method does not work for finding the root of the algebraic equation. After obtaining the roots of the algebraic equation using Birge-Vita method, the roots are corrected for the desired accuracy using the Newton-Raphson method. This combination has overcome the difficulties in finding the roots of the algebraic equations of the governing equations. In Fig.2 and Fig.3 the dispersion curve is drawn between the non dimensional circumferential wave number versus frequency of the cylindrical shell with respect to different rotational speed $\Omega = 0.2, 0.4, 0.6, 0.8$. In the present range of circumferential wave number it can be seen that the frequency parameter increases monotonically with increasing rotating speed. It shows the cylindrical shell rotating in high speed, the effect of the rotating speed on the frequency characteristics is more significant.

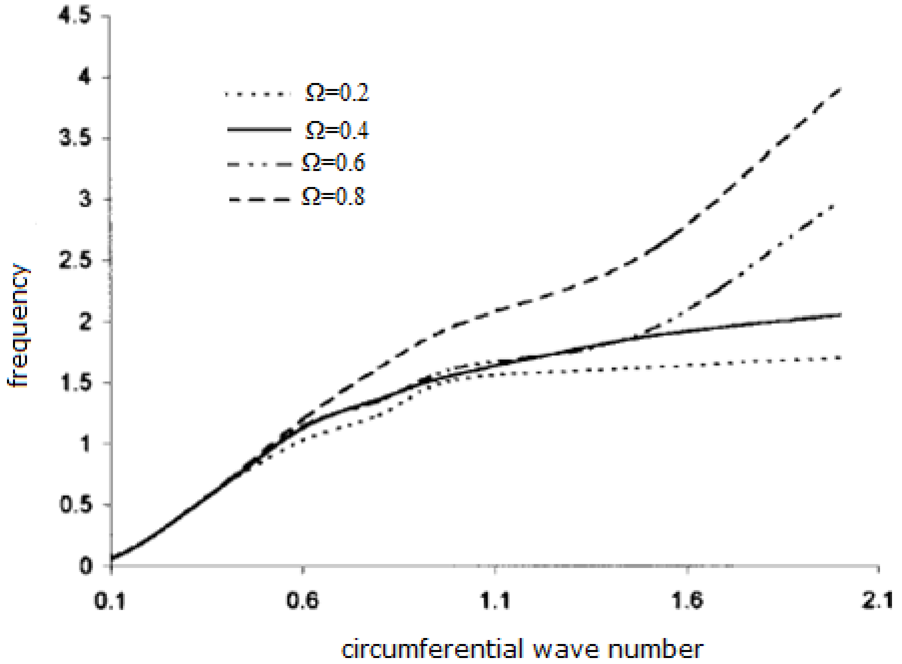

Fig.2.Variation of circumferential wave number verses frequency with different Ω for $t_L = 1$.

From the Figs.2 and 3, it is observed that the non-dimensional frequency increases rapidly to become dispersive at higher values of wave number for both $t_L = 1$ and $t_L = 2$ .The frequency of increasing value of Ω of rotating shell is observed to increase from zero wave number and become quite dispersive at higher values of wave number for both $t_L = 1$ and $t_L = 2$. When the rotational speed of the cylindrical panel is increased, the dimensionless frequency is also increases for both $t_L = 1$ and $t_L = 2$. The comparison of Fig.2 and Fig.3 shows that the non-dimensional frequency increases exponentially for smaller wave number in case of the axial wave number

$t_L = 1$ and $t_L = 2$ for all value of Ω, but the case of higher wave number the non-dimensional frequency is dispersive and slow for all values of Ω.

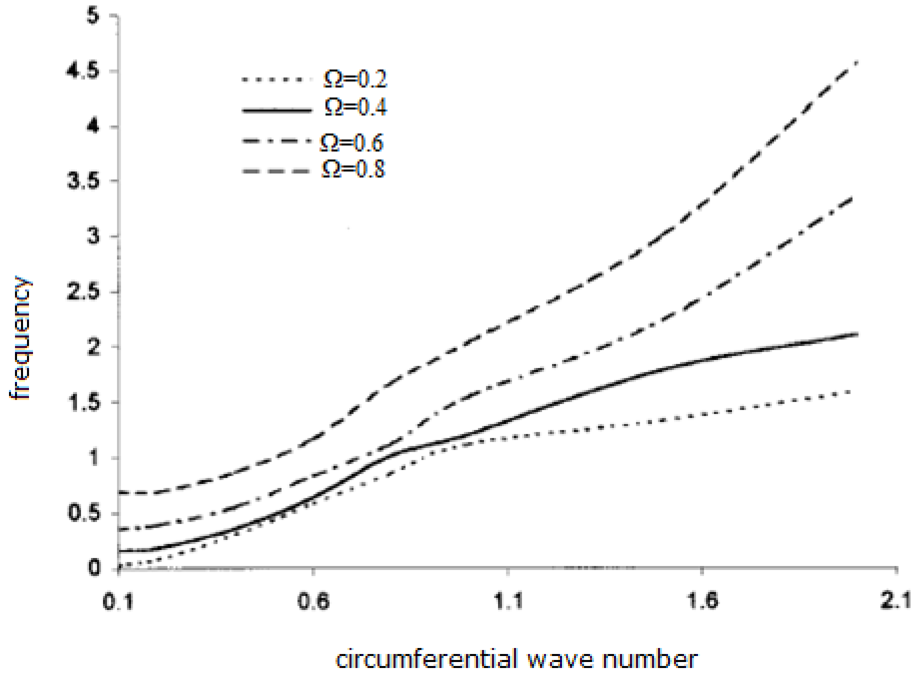

Fig.3.Variation of circumferential wave number verses frequency with different Ω for $t_L = 2$.

6. Conclusion

The three dimensional vibration analysis of a homogeneous isotropic rotating cylindrical panel subjected to the traction free boundary conditions has been considered for this paper. For this problem, the governing equations of three dimensional linear elasticity have been employed and solved by modified Bessel function with complex argument. The effect of the circumferential wave number on the frequency of a closed copper cylindrical shell is investigated and the results are presented as dispersion curves.

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