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Codes Detecting and Locating Solid Burst Errors

P. K. Das
Department of Mathematics
Shivaji College (University of Delhi)
Raja Garden, Delhi – 110 027, India
E-mail: pankaj4thapril@yahoo.co.in

Abstract Error-locating codes (EL codes), first proposed by J. K. Wolf and B. Elspas in 1963, have the potential to be used to identify the faulty module for fault isolation and reconfiguration in fault-tolerant computer systems. This paper proposes a new class of EL codes suitable for systems with *solid burst errors*. The author gives a necessary condition (see Theorem 1) and a sufficient condition (see Theorem 2) for the existence of linear codes that detect and locate errors which are in the form of *solid burst errors*. An illustration of such a code is also provided.

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1 Introduction

In coding theory, many types of error patterns have been dealt with and codes have been constructed to combat such error patterns. In certain memory systems (e.g. some spacecraft memories and supercomputer storage system), the most common errors are solid burst errors i.e. errors in bits which are stored physically adjacent in the memory. This motivates the interest in codes which deal with solid burst errors. A solid burst may be defined as follows:

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Definition 1 *A solid burst of length s is a vector with non zero entries in some s consecutive positions and zero elsewhere.*

In this paper we have presented a study of codes dealing with the location of such kind of errors occurring within a sub-block. The concept of error-locating codes, lying midway between error detection and error correction, was introduced by Wolf and Elspas [8]. In this technique the block of received digits is to be regarded as subdivided into mutually exclusive sub-blocks and while decoding it is possible to detect the error and in addition the receiver is able to identify which particular sub-block contains error. Such codes are referred to as Error-Locating codes (EL- codes). Wolf and Elspas [8] studied binary codes which are capable of detecting and locating a single sub-block containing random errors. Dass and Muttoo [5] improved the sufficient bound obtained by Wolf and Elspas. A study of such error locating codes in which errors occur in the form of bursts was made by Dass [1], then by Dass and Kishan Chand [3]. Similar study was also done by Dass and Arora [2], Dass and Madan [4] in which errors occur in the form of repeated bursts.

This paper gives a necessary condition for the detection and location of solid burst of length s or less in a linear code over GF(q). This is followed by a sufficient condition for the existence of such a code. An illustration of such code has also been given. Development of such codes will economize in the number of parity-check digits required in comparison to the usual solid burst of length s or less in the whole code length.

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2 Main Results

In what follows we shall consider a linear code C to be a proper subspace of n- tuples over GF(q). If G is a $k \times n$ generator matrix of the linear code C, the number of information digits is k and the number of parity check digits is $n-k$. If H is the matrix whose columns generate the null space of the code C, then the matrix H is called the parity check matrix of the code C. The quantity vH^T is called the syndrome of the vector v. The block of n digits, consisting of r parity check digits and $k = n - r$ information digits, is considered to be divided into b mutually exclusive sub-blocks. Each sub-block contains $t = n/b$ digits.

We consider (n, k) linear codes over GF(q) that are capable of detecting and locating all solid burst of length s or less in a single sub-block. It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of a solid burst of length s or less must satisfy the following conditions:

(a) The syndrome resulting from the occurrence of a solid burst of length s or less within any one sub-block must be distinct from the all zero syndrome.

(b) The syndrome resulting from the occurrence of any solid burst of length s or less within a single sub-block must be distinct from the syndrome resulting likewise from solid burst of length s or less within any *other* sub-block.

Now following is the first result that derives a lower bound on the number of check digits required for the existence of a linear code over GF(q) capable of detecting and locating a single sub-block containing errors that are solid burst of length s or less.

Theorem 1 *An* (n, k) *linear code over* GF*(q) that detects and locates a single corrupted* *sub-block containing errors that are in the form of solid burst of length s or less is divided into b mutually exclusive sub-blocks of length t each. The number of parity check digits r required for such an EL-code is bounded from below by*

$$r \geq \log_q[1 + b\sum_{i=1}^{s}(q-1)^i] \tag{1}$$

Proof. Let there be an (n, k) linear code vector GF(q) that detects and locates solid burst errors of length s or less within a single corrupted sub-block. The maximum number of distinct syndromes available using r check bits is q^r. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (a) and (b) and then setting this number less than or equal to q^r.

Let X be the set of all vectors whose non-zero components are in consecutive positions, confined to first s positions and the first position being non zero in any one sub block.

We claim that syndromes of all elements of X should be distinct; else for any x_1, x_2 in X having the same syndrome would imply that the syndrome of x_1-x_2 which is also an element of X and hence a syndrome of an solid burst error of length s or less within the same sub-block, becomes zero; in violation of condition (a).

Also, since the code locates a single sub-block containing solid burst of length s or less, the syndromes produced by the similar vectors in different sub- blocks must be distinct by condition (b). Thus, the syndromes of vectors which are solid burst of length s or less, whether in the same sub-block or in different sub-blocks, must be distinct.

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Now the number of vectors in X, excluding all zero vector, is

$$(q-1)+(q-1)^2+\cdots+(q-1)^s$$
$$=\sum_{i=1}^{s}(q-1)^i$$

As there are $\sum_{i=1}^{s}(q-1)^i$ distinct non-zero syndromes corresponding to vectors in any single sub-block and there are b sub-blocks in all, we must have at least

$$1+b\sum_{i=1}^{s}(q-1)^i$$

distinct syndromes, including the all zeros syndrome. Therefore, we must have

$$q^r \geq [1+b\sum_{i=1}^{s}(q-1)^i]$$

i.e.,

$$r \geq \log_q[1+b\sum_{i=1}^{s}(q-1)^i] \quad \square$$

In the following result, a lower bound on the number of check digits required for the existence of such a code is derived. The proof is based on the technique used to establish Varshamov-Gilbert-Sacks bound by constructing a parity check matrix for such a code (refer Sacks [7], also Theorem 4.17, Peterson and Weldon [6]). This technique not only ensures the existence of such a code but also gives a method for construction of the code.

Theorem 2 *An (n,k) linear EL code over* GF*(q), capable of detecting and locating any solid burst of length s or less occurring within a single sub-block, is divided into b mutually exclusive sub-blocks of length t each. There shall always exist such an EL code using r parity check digits provided that*

$$r > \log_q\left[\left\{1+\sum_{i=1}^{s-1}(q-1)^i\right\}\left\{1+(b-1)\sum_{i=1}^{s}(t-i+1)(q-1)^i\right\}\right]$$

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Proof. We shall prove the result by constructing an appropriate $(n-k)\times n$ parity check matrix H for the desired code. Suppose that the columns of the first $b-1$ sub-blocks of H and the first j - 1 columns h_1, h_2,..., h_{j-1} of the b^{th} sub-block have been appropriately added.

Now for the detection of solid burst of length s or less in the b^{th} sub-block, choose any non zero $(n-k)$ tuple h_1 as the first column of the b^{th} sub-block of the matrix H. After having selected the first $j-1$ columns $h_1, h_2, \ldots, h_{j-1}$ appropriately ,we lay down the condition to add j^{th} column h_j such that h_j should not be a linear sum of preceding consecutive up to $s-1$ columns. In other words

$$h_j \neq u_1 h_{j-1} + u_2 h_{j-2} + \cdots + u_{s'-2} h_{j-s'+2} + u_{s'-1} h_{j-s'+1} \tag{2}$$

where $s' \leq s$, $j \geq s'$, $u_i \in GF(\mathrm{q})$ and non zero u_i's occurs consecutively with u_1 being non zero.

This condition ensures that a solid burst of length s or less will not be a code word which there by means that the code shall be able to detect solid bursts of length s or less.

The number of linear sums on the right-hand side of (2), including the vector of all zeros, is

$$1 + \sum_{i=1}^{s-1} (q-1)^i \tag{3}$$

Thus, the total number of possible combinations different from h_j is given by (3).

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Further, for location of solid burst of length s or less, according to condition (b), the syndrome of any solid burst of length s or less within a sub-block must be different from the syndrome resulting from any solid burst of length s or less within any *other* sub-block. In view of this, h_j can be added provided that

$$\begin{aligned} h_j \quad &\neq (u_1 h_{j-1} + u_2 h_{j-2} + \cdots + u_{s'-2} h_{j-s'+2} + u_{s'-1} h_{j-s'+1}) \\ &+ (v_i h_{m+i} + v_{i+1} h_{m+i+1} + \cdots + v_{i+s''-1} h_{m+i+s''-1}) \end{aligned}$$

where s', s'' are less or equal to s, $j \geq s'$, $u_i, v_i \in GF(q)$, non zero u_i, v_i occur consecutively with u_1 being non zero and h_{m+i}'s are any consecutive s or less columns corresponding to m^{th} sub-block among the $b-1$ sub-blocks.

The number of ways in which the coefficients u_i's can be selected is given by (3).

To enumerate the coefficients v_i's is equivalent to enumerate the number of solid burst of length s or less in a vector of length t. This number of solid burst of length s or less within a sub-block of length t, excluding the vector of all zeros, is

$$\sum_{i=1}^{s} (t-i+1)(q-1)^i$$

Since there are $b-1$ previously chosen sub-blocks, therefore number of such linear combinations becomes

$$(b-1) \sum_{i=1}^{s} (t-i+1)(q-1)^i \tag{4}$$

So, for location, the number of linear combinations to which h_j can not be equal to is the

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product

$$\text{expr.}(3) \times \text{expr.}(4)$$

$$= \{1 + \sum_{i=1}^{s-1} (q-1)^i\}.(b-1) \sum_{i=1}^{s} (t-i+1)(q-1)^i \quad (5)$$

Thus, the total number of linear combinations different from h_j is the sum of linear combinations computed in (3) and (5). i.e.

$$\{1 + \sum_{i=1}^{s-1} (q-1)^i\} + \{1 + \sum_{i=1}^{s-1} (q-1)^i\}(b-1) \sum_{i=1}^{s} (t-i+1)(q-1)^i$$

At worst, all these combinations might yield a distinct sum. Therefore, h_j can be added to the b^{th} sub-block of H provided that

$$q^r > \left\{1 + \sum_{i=1}^{s-1} (q-1)^i\right\}\left\{1 + (b-1) \sum_{i=1}^{s} (t-i+1)(q-1)^i\right\}$$

Thus for completing the b^{th} sub block,

$$r > \log_q \left[\left\{1 + \sum_{i=1}^{s-1} (q-1)^i\right\}\left\{1 + (b-1) \sum_{i=1}^{s} (t-i+1)(q-1)^i\right\}\right] \quad \square$$

The paper is concluded with an example and acknowledgement.

Example 1 Consider a $(9,3)$ binary code with the 6×9 matrix H which has been constructed by the synthesis procedure given in the proof of theorem 2 by taking $s = 3, t = 3, b = 3$.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The null space of this matrix can be used as a code to detect and locate solid bursts of length 3 or less within a sub-block of length $t = 3$. It may be easily verified from error pattern-syndromes table 1 that all the syndromes are non zero and distinct.

Table 1

Error patterns	Syndromes
1^{st} sub-block	
100 000 000	100000
010 000 000	010000
001 000 000	001000
110 000 000	110000
011 000 000	011000
111 000 000	1110000
2^{nd} sub-block	
000 100 000	000100
000 010 000	000010
000 001 000	000001
000 110 000	000110
000 011 000	000011
000 111 000	000111
3^{rd} sub-block	
000 000 100	100001
000 000 010	010010
000 000 001	001100
000 000 110	110011
000 000 011	011110
000 000 111	111111

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