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ON NEW CESÀRO-ORLICZ DOUBLE DIFFERENCE SEQUENCE SPACE

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ABSTRACT. The aim of this paper is to introduce the Cesàro-Orlicz double difference sequence space $Ces_M^{(2)}(\triangle, p)$. We study some topological properties of this space and give some inclusion relations.

1. INTRODUCTION

Throughout this work, $\mathbb{N}$, $\mathbb{R}$, w and w^2 denote the sets of positive integers, real numbers, single real sequences and double real sequences, respectively.

First of all, let us recall preliminary definitions and notations.

A double sequence on a normed linear space X is a function x from $\mathbb{N}\times\mathbb{N}$ into X and briefly denoted by $x=(x_{kl})$. If for every $\varepsilon>0$ there exists $n_\varepsilon\in\mathbb{N}$ such that $\|x_{kl}-a\|_X<\varepsilon$ whenever $k,l>n_\varepsilon$ then a double sequence (x_{kl}) is said to be converges (in terms of Pringsheim) to $a\in X$ [16].

A double series $\sum_{k,l=1}^{\infty}x_{kl}$ is convergent if and only if its sequence of partial sums (s_{nm}) is convergent (see [2],[3]), where $s_{nm}=\sum_{k=1}^{n}\sum_{l=1}^{m}x_{kl}$ for all $m,n\in\mathbb{N}$.

Let X be a linear space. A function $p:X\to\mathbb{R}$ is called *paranorm*, if

i) $p(0)=0$,

ii) $p(x)\geq 0$ for all $x\in X$,

iii) $p(-x)=p(x)$ for all $x\in X$,

iv) $p(x+y)\leq p(x)+p(y)$ for all $x,y\in X$,

v) if (λ_n) is a sequence of scalars with $\lambda_n\to\lambda$ as $n\to\infty$ and (x_n) is a sequence of vectors with $p(x_n-x)\to 0$ as $n\to\infty$, then $p(\lambda_n x_n-\lambda x)\to 0$ as $n\to\infty$ (continuity of scalars multiplication).

A paranorm p for which $p(x)=0$ implies $x=0$ is called *total* [12].

An Orlicz function is a function $M:[0,\infty)\to[0,\infty)$ which is continuous, non-decreasing and convex with $M(0)=0$, $M(x)>0$ for $x>0$ and $M(x)\to\infty$ as $x\to\infty$.

An Orlicz function M can always be represented in the following integral form: $M(x)=\int\limits_0^x\eta(t)\,dt$, where η is known as the kernel of M, is right differentiable for $t\geq 0$, $\eta(0)=0$, $\eta(t)>0$ for $t>0$, η is nondecreasing and $\eta(t)\to\infty$ as $t\to\infty$.

An Orlicz function M is said to be satisfied the Δ_2-condition if there are $T>0$ and $a>0$ such that $M(a)>0$ and $M(2u)\leq TM(u)$ for all $u\in[0,a]$ (see [10]).

For $1\leq p<\infty$, the Cesàro sequence space Ces_p is defined by

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$$Ces_p = \left\{ x \in w : \sum_{j=1}^{\infty} \left(\frac{1}{j} \sum_{i=1}^{j} |x_i| \right)^p < \infty \right\},$$

equipped with norm

$$\|x\| = \left(\sum_{j=1}^{\infty} \left(\frac{1}{j} \sum_{i=1}^{j} |x_i| \right)^p \right)^{\frac{1}{p}}.$$

This space was first introduced by Shiue [18]. It is very useful in the theory of matrix operators and others. Sanhan and Suantai introduced and studied a generalized Cesàro sequence space $Ces(p)$, where $p = (p_j)$ is a bounded sequence of positive real numbers (see [17]). Later, this space was studied by many authors in [4], [7], [9], [11], [14], [15].

The notion of difference sequence space was introduced by Kızmaz in [8] in 1981 as follows:

$$X(\triangle) = \{x = (x_k) \in w : (x_k - x_{k+1}) \in X\}$$

for $X = \ell_\infty,\ c,\ c_0$. Subsequently difference sequence spaces has been discussed in Ahmad and Mursaleen [1], Malkowsky and Parashar [13], Et and Başarır [5], Et and Çolak [6] and others.

In this work, we introduce double sequence spaces $Ces_M^{(2)}(\triangle, p)$ as follows;

Let $p = (p_{nm})$ be a bounded double sequence of positive real numbers and M be an Orlicz function. The space $Ces_M^{(2)}(\triangle, p)$ is defined by

$$Ces_M^{(2)}(\triangle, p) = \left\{ x \in w^2 : \sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty, \exists \rho > 0 \right\},$$

where $\triangle x_{ij} = x_{i-1,j-1} - x_{i-1,j} - x_{i,j-1} + x_{ij}$ for all $i, j \in \mathbb{N}$ and the terms with negative subscript are assume zero.

The following inequality will be used throughout this paper. Let (p_{nm}) be a bounded double sequence of strictly positive real numbers and denote $H = \sup_{n,m} p_{nm}$. For any complex a_{nm} and b_{nm} we have

$$|a_{nm} + b_{nm}|^{p_{nm}} \leq D. \left(|a_{nm}|^{p_{nm}} + |b_{nm}|^{p_{nm}} \right)$$

where $D = \max\left(1, 2^{H-1}\right)$. Also, for any complex λ,

$$|\lambda|^{p_{nm}} \leq \max\left(1, |\lambda|^H\right).$$

2. MAIN RESULTS

Theorem 1. *Let (p_{nm}) be bounded. The set $Ces_M^{(2)}(\triangle, p)$ of double sequences is a linear space over the real field $\mathbb{R}$.*

Proof. Let $x, y \in Ces_M^{(2)}(\triangle, p)$ and $\lambda, \beta \in \mathbb{C}$. Then there exist $\rho_1 > 0, \rho_2 > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho_1} \right) \right]^{p_{nm}} < \infty$$

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and

$$\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}<\infty.$$

Let $\alpha,\beta\in\mathbb{R}$ and $\rho_3=\max\{2|\alpha|\rho_1,2|\beta|\rho_2\}$. Since M is non-decreasing convex function, we have

$$\begin{aligned}
&\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\alpha\triangle x_{ij}+\beta\triangle y_{ij}|}{\rho_3}\right)\right]^{p_{nm}}\\
\leq\;&\sum_{n,m=1}^{\infty}\left[M\left(\frac{|\alpha|}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_3}+\frac{|\beta|}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_3}\right)\right]^{p_{nm}}\\
\leq\;&\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{2nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}+\frac{1}{2nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\\
\leq\;&\sum_{n,m=1}^{\infty}\frac{1}{2^{p_{nm}}}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)+M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\\
<\;&\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)+M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\\
\leq\;&D.\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right]^{p_{nm}}\\
&+D.\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\\
<\;&\infty,
\end{aligned}$$

where $D=\max\left(1,2^{H-1}\right)$. This shows that $\lambda x+\beta y\in Ces_M^{(2)}(\triangle,p)$ and so $Ces_M^{(2)}(\triangle,p)$ is a linear space. □

Theorem 2. *The double sequence space $Ces_M^{(2)}(\triangle,p)$ is a paranormed space with the paranorm*

$$g(x)=\inf\left\{\rho^{\frac{p_{qr}}{R}}>0:\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\leq 1,q,r\in\mathbb{N}\right\}$$

where $H=\sup_{n,m}p_{nm}<\infty$ and $R=\max(1,H)$.

Proof. It is clear that $g(x)=g(-x)$ and $g(0)=0$. For any $x,y\in Ces_M^{(2)}(\triangle,p)$, there exist $\rho_1,\rho_2>0$ such that

$$\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\leq 1$$

and

$$\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\leq 1.$$

Let $\rho_3=2^{\frac{R}{h}}(\rho_1+\rho_2)$, where $h=\inf p_{nm}>0$. Since M is a non-decreasing convex function, we have

$$\begin{aligned}
&\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}+\triangle y_{ij}|}{\rho_3}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
\leq\ &\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{2^{\frac{R}{h}}(\rho_1+\rho_2)}+\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{2^{\frac{R}{h}}(\rho_1+\rho_2)}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
\leq\ &\left(\sum_{n,m=1}^{\infty}\left[\frac{\rho_1}{2^{\frac{R}{h}}(\rho_1+\rho_2)}M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right.\right.\\
&\left.\left.+\frac{\rho_2}{2^{\frac{R}{h}}(\rho_1+\rho_2)}M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
\leq\ &\left(\sum_{n,m=1}^{\infty}\left[\frac{1}{2^{\frac{R}{h}}}M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
&+\left(\sum_{n,m=1}^{\infty}\left[\frac{1}{2^{\frac{R}{h}}}M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
=\ &\frac{1}{2}\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
&+\frac{1}{2}\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\\
\leq\ &1.
\end{aligned}$$

Since ρ_1,ρ_2,ρ_3 are positive real numbers we get

$$\begin{aligned}
g(x+y)\ =\ &\inf\left\{\rho_3^{\frac{pqr}{R}}>0:\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}+\triangle y_{ij}|}{\rho_3}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\right.\\
&\left.\leq 1;q,r\in\mathbb{N}\right\}
\end{aligned}$$

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$$
\begin{aligned}
&\leq \inf\left\{\rho_1^{\frac{p_{qr}}{R}}>0:\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho_1}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\leq 1;q,r\in\mathbb{N}\right\}\\
&\quad+\inf\left\{\rho_2^{\frac{p_{qr}}{R}}>0:\left(\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle y_{ij}|}{\rho_2}\right)\right]^{p_{nm}}\right)^{\frac{1}{R}}\leq 1;q,r\in\mathbb{N}\right\}\\
&= g(x)+g(y).
\end{aligned}
$$

Let $(x^n)=\left\{x_{ij}^n\right\}$ be any sequence in the space $Ces_M^{(2)}(\triangle,p)$ such that $g(x^n-x)\to 0$, as $n\to\infty$ and (λ_n) is a sequence of reals with $\lambda_n\to\lambda$, as $n\to\infty$. Then, since the inequality

$$g(x^n)\leq g(x)+g(x^n-x)$$

holds by subadditivity of the function g, $\{g(x^n)\}$ is bounded. Taking into account this fact we therefore derive the inequality

$$g(\lambda_n x^n-\lambda x)\leq|\lambda_n-\lambda|\,g(x^n)+|\lambda|\,g(x^n-x)$$

which tends to zero as $n\to\infty$. Hence, the scalar multiplication is continuous.

That is to say that g is a paranorm on the space $Ces_M^{(2)}(\triangle,p)$, as asserted. □

Theorem 3. *The space $Ces_M^{(2)}(\triangle,p)$ is complete with respect to its paranorm.*

Proof. Let $(x^s)=\left\{x_{ij}^s\right\}$ be any Cauchy sequence in the space $Ces_M^{(2)}(\triangle,p)$. Since (x^s) is a Cauchy sequence, we have

$$g\left(x^s-x^t\right)\to 0 \tag{1}$$

as $s,t\to\infty$. Hence, we get

$$\left|\triangle x_{ij}^s-\triangle x_{ij}^t\right|\to 0$$

as $s,t\to\infty$ for all $i,j\in\mathbb{N}$. Then, we have $\left\{x_{ij}^s\right\}$ is a Cauchy sequence in $\mathbb{R}$ for each fixed $i,j\in\mathbb{N}$.

Thus, there exists $x_{ij}\in\mathbb{R}$ such that $x_{ij}^s\to x_{ij}$ as $s\to\infty$ and say $x=x_{ij}$. Since M is continuous, by (1) we get

$$g\left(x^s-x\right)\to 0$$

as $t\to\infty$.

Since $Ces_M^{(2)}(\triangle,p)$ is linear space, we get $x=\{x_{ij}\}\in Ces_M^{(2)}(\triangle,p)$. This completes the proof. □

Theorem 4. *Let $0<p_{nm}\leq q_{nm}<\infty$. Then $Ces_M^{(2)}(\triangle,p)\subset Ces_M^{(2)}(\triangle,q)$.*

Proof. Let $x\in Ces_M^{(2)}(\triangle,p)$, then there exists $\rho>0$ such that

$$\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho}\right)\right]^{p_{nm}}<\infty.$$

Hence we have $M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho}\right)<1$ for large values of n,m. Then, we get

$$\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho}\right)\right]^{q_{nm}}\leq\sum_{n,m=1}^{\infty}\left[M\left(\frac{1}{nm}\sum_{i,j=1}^{n,m}\frac{|\triangle x_{ij}|}{\rho}\right)\right]^{p_{nm}}<\infty$$

and so $x \in Ces_M^{(2)}(\triangle, q)$. $\square$

Theorem 5. *Let M_1 and M_2 be Orlicz functions satisfying $\triangle_2-$condition. Then*

(a) $Ces_{M_1}^{(2)}(\triangle, p) \subset Ces_{M_2 \circ M_1}^{(2)}(\triangle, p)$,
(b) $Ces_{M_1}^{(2)}(\triangle, p) \cap Ces_{M_2}^{(2)}(\triangle, p) \subset Ces_{M_1+M_2}^{(2)}(\triangle, p)$.

Proof. (a) Let $x \in Ces_{M_1}^{(2)}(\triangle, p)$. Then there exists $\rho > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty.$$

Since M_1 is a continuous function, we can find a real number δ with $0 < \delta < 1$ such that $M_1(t) < \varepsilon$. Let $y_{nm} = M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right)$. Hence we write

$$\sum_{n,m=1}^{\infty} [M_2(y_{nm})]^{p_{nm}} = \sum_{y_{nm} \leq \delta}^{\infty} [M_2(y_{nm})]^{p_{nm}} + \sum_{y_{nm} > \delta}^{\infty} [M_2(y_{nm})]^{p_{nm}}.$$

By the properties of M_2, we have

$$\sum_{y_{nm} \leq \delta}^{\infty} [M_2(y_{nm})]^{p_{nm}} \leq \max\left\{1, M_2(1)^H\right\} \sum_{y_{nm} \leq \delta}^{\infty} [y_{nm}]^{p_{nm}}. \tag{2}$$

Also,

$$M_2(y_{nm}) < M_2\left(1 + \frac{y_{nm}}{\delta}\right) < \frac{1}{2} M_2(2) + \frac{1}{2}\left(\frac{2y_{nm}}{\delta}\right)$$

for $y_{nm} > \delta$. Since M_2 satisfying $\triangle_2-$condition and $\frac{y_{nm}}{\delta} > 1$, there exists $T > 0$ such that

$$M_2(y_{nm}) < \frac{1}{2} T \frac{y_{nm}}{\delta} M_2(2) + \frac{1}{2} T \frac{y_{nm}}{\delta} M_2(2) = T \frac{y_{nm}}{\delta} M_2(2).$$

Therefore we have

$$\sum_{y_{nm} > \delta}^{\infty} [M_2(y_{nm})]^{p_{nm}} \leq \max\left\{1, \left(T \frac{M_2(2)}{\delta}\right)^H\right\} \sum_{y_{nm} > \delta}^{\infty} [y_{nm}]^{p_{nm}}. \tag{3}$$

Hence by the $(2), (3)$, we get

$$\begin{aligned} \sum_{n,m=1}^{\infty} \left[(M_2 \circ M_1) \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right]^{p_{nm}} &= \sum_{n,m=1}^{\infty} [M_2(y_{nm})]^{p_{nm}} \\ &\leq B \cdot \sum_{y_{nm} \leq \delta}^{\infty} [y_{nm}]^{p_{nm}} \\ &\quad + F \cdot \sum_{y_{nm} > \delta}^{\infty} [y_{nm}]^{p_{nm}} \\ &< \infty, \end{aligned}$$

where $B = \max\left\{1, M_2(1)^H\right\}$ and $F = \max\left\{1, \left(T \frac{M_2(2)}{\delta}\right)^H\right\}$. Hence $Ces_{M_1}^{(2)}(\triangle, p)$ $\subset Ces_{M_2 \circ M_1}^{(2)}(\triangle, p)$.

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(b) Let $x \in Ces_{M_1}^{(2)}(\triangle, p) \cap Ces_{M_2}^{(2)}(\triangle, p)$. Then there exists $\rho > 0$ such that

$$\sum_{n,m=1}^{\infty} \left[M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty$$

and

$$\sum_{n,m=1}^{\infty} \left[M_2 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right]^{p_{nm}} < \infty.$$

Hence we get

$$\begin{aligned} & \sum_{n,m=1}^{\infty} \left((M_1 + M_2) \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ \leq \quad & A \cdot \sum_{n,m=1}^{\infty} \left(M_1 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ & + A \cdot \sum_{n,m=1}^{\infty} \left(M_2 \left(\frac{1}{nm} \sum_{i,j=1}^{n,m} \frac{|\triangle x_{ij}|}{\rho} \right) \right)^{p_{nm}} \\ < \quad & \infty, \end{aligned}$$

where $A = \max\left\{1, 2^{H-1}\right\}$. This completes the proof. $\square$

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