ROMANIAN JOURNAL OF MATHEMATICS AND COMPUTER SCIENCE, 2016, VOLUME 6, ISSUE 1, p.47-60

A MATHEMATICAL MODELING OF CAMPUS INFORMATION SYSTEM

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Abstract. An H-magic labeling in a H-decomposable graph G is a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that for every copy H in the decomposition, $\sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e)$ is constant. f is said to be H-V-super magic if $f(V(G)) = \{1, 2, ..., p\}$. Suppose that $V(G) = U(G) \cup W(G)$ with $|U(G)| = m$ and $|W(G)| = n$. Then f is said to be H-V-super-strong magic labeling if $f(U(G)) = \{1, 2, ..., m\}$ and $f(W(G)) = \{m+1, m+2, ..., (m+n = p)\}$. A graph that admits a H-V-super-strong magic labeling is called a H-V-super-strong magic decomposable graph. In this paper, we pay our attention to provide a mathematical modeling of campus information system.

Mathematics Subject Classification (2010): $05C78$, $05C70$.
Keywords: H-decomposable graph, H-V-super-strong magic labeling, complete bipartite graph.

Article history:
Received 4 December 2015
Received in revised form 1 June 2016
Accepted 3 June 2016

1. Introduction

In this paper we consider only finite and simple undirected bipartite graphs. The vertex and edge sets of a graph G are denoted by $V(G)$ and $E(G)$ respectively and we let $|V(G)| = p$ and $|E(G)| = q$. For graph theoretic notations, we follow [3], [4]. A labeling of a graph G is a mapping that carries a set of graph elements, usually vertices and/or edges into a set of numbers, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [7].

Although magic labeling of graphs was introduced by Sedlacek [14], the concept of vertex magic total labeling (VMTL) first appeared in 2002 in [8]. In 2004, MacDougall et al. [9] introduced the notion of super vertex magic total labeling (SVMTL). In 1998, Enomoto et al. [5] introduced the concept of super edge-magic graphs. In 2005, Sugeng and Xie [17] constructed some super edge-magic total graphs. The usage of the word "super" was introduced in [5]. The notion of a V-super vertex magic labeling was introduced by MacDougall et al. [9] as in the name of super vertex-magic total labeling and it was renamed as V-super vertex magic labeling by Marr and Wallis in [12] after referencing the article [10].

A vertex magic total labeling is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, ..., p+q$ with the property that for every $u \in V(G)$, $f(u) + \sum_{v\in N(u)} f(uv) = k$ for some constant k, such a labeling is V-super if $f(V(G)) = \{1, 2, ..., p\}$. A graph G is called V-super vertex magic if it admits a V-super vertex labeling. A vertex magic total labeling is called E-super if $f(E(G)) = \{1, 2, ..., q\}$. A graph G is called E-super vertex magic if it admits a E-super vertex labeling. The results of the article [10] can also be found in [12]. In [9], MacDougall et al., proved that no complete bipartite graph is V-super vertex magic. An edge-magic total labeling is a bijection f from $V(G) \cup E(G)$ to the integers $1, 2, ..., p + q$ with the property that for any edge $uv \in E(G)$, $f(u) + f(uv) + f(v) = k$ for some constant k, such a labeling is super if $f(V(G)) = \{1, 2, ..., p\}$. A graph G is called super edge-magic if it admits a super edge-magic labeling.

Recently, Marimuthu and Balakrishnan [11], introduced the notion of super edge-magic graceful graphs to solve some kind of network problems. A (p, q) graph G with p vertices and q edges is edge magic graceful if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that $|f(u) + f(v) - f(uv)| = k$, a constant for any edge uv of G. G is said to be super edge-magic graceful if $f(V(G)) = \{1, 2, ..., p\}$.

A family of subgraphs $H_1, H_2, \cdots, H_h$ of G is a H-decomposition of G if all the subgraphs are isomorphic to a graph H, $E(H_i) \cap E(H_j) = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{h} E(H_i) = E(G)$. In this case, we write $G = H_1 \oplus H_2 \oplus \cdots \oplus H_h$ and G is said to be H-decomposable. In [16], Subbiah and Pandimadevi introduced the notion of H-E-super magic decomposition of graphs. In [15], Stalin Kumar and Marimuthu studied the H-V-super magic decomposition of complete bipartite graphs. Suppose G is H-decomposable. A total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p + q\}$ is called an H-magic labeling of G if there exists a positive integer k (called magic constant) such that for every copy H in the decomposition, $\sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e) = k$. A graph G that admits such a labeling is called a H-magic decomposable graph. An H-magic labeling f is called a H-V-super magic labeling if $f(V(G)) = \{1, 2, \cdots, p\}$. A graph that admits a H-V-super magic labeling is called a H-V-super magic decomposable graph. An H-magic labeling f is called a H-E-super magic labeling if $f(E(G)) = \{1, 2, \cdots, q\}$. A graph that admits a H-E-super magic labeling is called a H-E-super magic decomposable graph. The sum of all vertex and edge labels on H, under a labeling f is denoted by $\sum f(H)$.

In many of the results about H-magic decomposable graph or H-V-super magic decomposable graph or H-E-super magic decomposable graphs, the host graph G is required to be H-decomposable. Yoshimi Ecawa et al [18] studied the decomposition of complete bipartite graphs into edge-disjoint subgraphs with star components. The notion of star-subgraph was introduced by Akiyama and Kano in [1]. A subgraph F of a graph G is called a star-subgraph if each component of F is a star. Here by a star, we mean a complete bipartite graph of the form $K_{1,m}$ with $m \geq 1$. A subgraph F of a graph G is called a n-star-subgraph if $F \cong K_{1,n}$ with $2 \leq n < p$. Marimuthu and Stalin Kumar [15] studied about the H-V-super magic decomposition of complete bipartite graph $K_{n,n}$. It is difficult to study the H-V-super magic decomposition and H-E-super magic decomposition of complete bipartite graphs $K_{m,n}$ with $m \neq n$. This helps us to have a detailed study of H-V-super-strong magic decomposition $K_{m,n}$.

In 2001, Muntaner-Batle [13] introduced the concept of super-strong magic labeling of bipartite graph as in the name of special super magic labeling of bipartite graph and it was renamed as super-strong magic labeling by Marr and Wallis [12]. Suppose G is a bipartite graph with vertex sets V_1 and V_2 of sizes m and n respectively. An edge-magic total labeling of G is super-strong if the elements of V_1 receive labels $\{1, 2, ..., m\}$ and the elements of V_2 receive labels $\{m + 1, m + 2, ..., m + n\}$.

Star topology is one of the most common network setups where each of the devices or nodes on a network connects to a central hub, switch or computer; the hub acting as a server and the peripheral devices as clients. For different network topologies, we refer to [2], [6]. A n-star topology is a network setup with n clients connected to a central hub.

Campus information system (CIS) is an interrelated group of information sources, accessible by computer through campus institutional external and internal web environment, that a institution places at the disposal of its users to enable them to consult it and/or provide a selection of significant relevant data, in the wide context of their institution life in its academic, administrative and social senses, in

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order to improve students knowledge base. It intends to provide the user with various selected sets of data that deals with the institution. In some cases, the system can enable with other people in institution in order to enhance various information exchange or knowledge sharing.

We look at a computer network as a connected undirected graph. A network designer may want to retrieve the data stored by a client through any server on the failure of any number of servers such that the performance of all servers are same. To achieve this one must consider the following assumptions.

(1) There are m servers and n clients. Each server and each client will have a unique preference number.
(2) No two servers are connected. No two clients are connected
(3) Each client is connected with all servers and will have an unique distance for each connection.
(4) Data entered by any client will be stored in all the servers.
(5) There are m n-star topologies.
(6) Failure of any of the servers will not affect the data stored in other servers.
(7) Data can be retrieved from any server through any client based on retrievers preference number given to the servers.

Let us consider the following generalization of this problem. Let the set $A = \{u_1, u_2, \cdots, u_m\}$ be the collection of m servers and the set $B = \{v_1, v_2, \cdots, v_n\}$ be the collection of n clients. The graph G with $V(G) = A \cup B$ and $E(G) = \{(u_i, v_j)|u_i \in A, v_j \in B, 1 \leq i \leq m, 1 \leq j \leq n\}$ denotes a complete bipartite graph $K_{m,n}$. Let $H \cong K_{1,n}$ denotes a n-star and $H_1, H_2, \cdots, H_m$ be a H-decomposition of G. Since the performance of all servers are same, so we want to find a bijection $f : V(G) \cup E(G) \to \{1, 2, \cdots, p + q(m + n + q)\}$ such that for every copy H in the decomposition, $\sum_{v\in V(H)} f(v) + \sum_{e\in E(H)} f(e) = k$ with $f(A) = \{1, 2, ..., m\}$, $f(B) = \{m + 1, m + 2, ..., (m + n = p)\}$ and $f(E(G)) = \{p + 1, p + 2, \cdots, p + q\}$. This motivates, the introduction of H-V-super-strong magic decomposition of $K_{m,n}$ and H-E-super-strong magic decomposition of complete bipartite graphs $K_{m,n}$.

Suppose G is H-decomposable and if $V(G) = U(G) \cup W(G)$ with $|U(G)| = m$ and $|W(G)| = n$. An H-V-super magic labeling f is called a H-V-super-strong magic labeling if $f(U(G)) = \{1, 2, ..., m\}$ and $f(W(G)) = \{m + 1, m + 2, ..., m + n\}$. A graph that admits a H-V-super-strong magic labeling is called a H-V-super-strong magic decomposable graph. An H-E-super magic labeling f is called a H-E-super-strong magic labeling if $f(U(G)) = \{q + 1, q + 2, ..., q + m\}$ and $f(W(G)) = \{q + m + 1, q + m + 2, ..., q + m + n\}$. A graph that admits a H-E-super-strong magic labeling is called a H-E-super-strong magic decomposable graph.

2. CIS and H-V-super-strong magic decomposable graph

Students information entered by different offices of the institution are stored in all the servers of the institution. The data stored can be viewed from any server by any client or office or user through their unique id, based on the preference number of the servers provided by the software company.

Let us label the m-servers by $\{1, 2, \cdots, m\}$ and the n-clients by $\{m+1, m+2, \cdots, m+n\}$. The set of all labels from server i to client j is $\{m+n+1, m+n+2, \cdots, m+n+q\}$, where $1 \leq i \leq m$ and $1 \leq j \leq n$. The n-star decomposition of the above problem together with H-V-super-strong magic labeling can be used to view data from any server on the failure of any number of servers with the same performance, if $k = \sum f(H_1) = \sum f(H_2) = \cdots = \sum f(H_m)$, where k is called the magic constant. In the next section we find the necessary and sufficient condition for a graph to be H-V-super-strong magic decomposable.

3. A necessary and sufficient condition

In this section, we consider the graphs $G \cong K_{m,n}$ and $H \cong K_{1,n}$, where $n \geq 1$. Clearly $p = m + n$ and $q = mn$.

Table 1. The edge label of a n-star-decomposition of G if m is even and n is odd.

f	v_1	v_2	...	v_{n-1}	v_n
u_1	$(m+n)$ $+m$	$(2m+n)$ $+1$	...	$(m+n)$ $+((n-2)m+1)$	$(m+n)$ $+mn$
u_2	$(m+n)+$ $(m-1)$	$(2m+n)$ $+2$	...	$(m+n)$ $+((n-2)m+2)$	$(m+n)$ $+(mn-1)$
u_3	$(m+n)+$ $(m-2)$	$(2m+n)$ $+3$	...	$(m+n)$ $((n-2)m+3)$	$(m+n)$ $+(mn-2)$
$\vdots$	...	...	...	...	...
u_k	$(m+n)+$ $(m-(k-1))$	$(2m+n)$ $+k$	...	$(m+n)$ $+((n-2)m+k)$	$(m+n)+(n-1)m$ $+(m-(k-1))$
$\vdots$	...	...	...	...	...
u_{m-1}	$(m+n)+$ 2	$(2m+n)$ $+(m-1)$	...	$(m+n)$ $+((n-2)m+(m-1))$	$(m+n)$ $+((n-1)m+2)$
u_m	$(m+n)+$ 1	$(2m+n)$ $+m$	...	$(m+n)+1$ $+((n-2)m+m)$	$(m+n)$ $+((n-1)m+1)$

The following theorem is useful in finding the magic constant k.

Theorem 3.1. *Suppose $\{H_1, H_2, \cdots, H_m\}$ is a H-decomposition of G with at least m or n is odd. Then G is H-V-super-strong magic decomposable with magic constant $k = (mn+1) + \frac{(n+1)(n(m+3)+(m-1))}{2}$.*

Proof. Let $U = \{u_1, u_2, \cdots, u_m\}$ and $V = \{v_1, v_2, \cdots, v_n\}$ be two stable sets of G. Let $\{H_1, H_2, \cdots, H_m\}$ be a n-star decomposition of G, where each H_i is isomorphic to H, such that $V(H_i) = \{u_i, v_1, v_2, \cdots, v_n\}$ and $E(H_i) = \{u_iv_1, u_iv_2, \cdots, u_iv_n\}$, for all $1 \le i \le m$. Define a total labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ by $f(u_i) = i$ and $f(v_j) = m + j$, for all $1 \le i \le m$ and $1 \le j \le n$.

Case 1: m is even and n is odd.
Now the edges of G can be labeled as shown in Table 1.
We prove the result for $n = k$ and the result follows for all $1 \le k \le m$.
From Table 1 and from definition of f, we get

$$\begin{aligned}\sum f(H_k) &= f(u_k) + \sum_{i=1}^{n} f(v_i) + \sum_{i=1}^{n} f(u_kv_i) \\ &= k + \sum_{i=1}^{n}(m+i) + \sum_{i=1}^{n} f(u_kv_i).\end{aligned}$$

Now,

$$\begin{aligned}\sum_{i=1}^{n} f(v_i) &= (m+1) + (m+2) + \cdots + (m+n) \\ &= mn + (1+2+\cdots+n) \\ &= mn + \frac{n(n+1)}{2}.\end{aligned}$$

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Also

$$\begin{aligned}
\sum_{i=1}^{n} f(u_k v_i) &= ((m+n)+(m-(k-1)))+((m+n)+(m+k))+\cdots \\
&\quad +((m+n)+(n-2)m+k)+((m+n)+(n-1)m+(m-(k-1))) \\
&= (2m+(n-(k-1)))+(2m+(n+k))+(4m+(n-(k-1)))+ \\
&\quad (4m+(n+k))+\cdots+((n-1)m+(n+k))+ \\
&\quad ((n+1)m+(n-(k-1))) \\
&= 2(2m+4m+\cdots+(n-1)m)+n(n)+\frac{n-1}{2}(1) \\
&\quad +((n+1)m-(k-1)) \\
&= 4m(1+2+\cdots+\frac{n-1}{2})+n^2+\frac{n-1}{2}+nm+m-(k-1) \\
&= 4m(\frac{(n-1)(n+1)}{8})+n^2+\frac{n-1}{2}+nm+m-(k-1) \\
&= \frac{m(n^2-1)}{2}+n^2+\frac{n-1}{2}+nm+m-(k-1) \\
&= \frac{mn^2-m+2n^2+n-1+2nm+2m}{2}-(k-1) \\
&= \frac{(mn^2+2nm+m)+(2n^2+n-1)}{2}-(k-1) \\
&= \frac{m(n+1)^2+(2n-1)(n+1)}{2}-(k-1).
\end{aligned}$$

Using the above values, we get

$$\begin{aligned}
\sum f(H_k) &= k+mn+\frac{n(n+1)}{2}+\frac{m(n+1)^2+(2n-1)(n+1)}{2}-(k-1) \\
&= (mn+1)+\frac{(n+1)(n+m(n+1)+(2n-1))}{2} \\
&= (mn+1)+\frac{(n+1)(n(m+3)+(m-1))}{2}.
\end{aligned}$$

Thus in this case the graph G is a H-V-super-strongly magic decomposable graph.

Case 2: m is odd and n is even.
Now the edges of G can be labeled as shown in Table 2.

Subcase(i): i is odd, where $1 \leq i \leq m$.
We prove the result for $i = j$ and the result follows for all $1 \leq i \leq m$ and i is odd.
From Table 2 and from definition of f, we get

$$\begin{aligned}
\sum f(H_j) &= f(u_j)+\sum_{i=1}^{n} f(v_i)+\sum_{i=1}^{n} f(u_j v_i) \\
&= j+mn+\frac{n(n+1)}{2}+\sum_{i=1}^{n} f(u_j v_i).
\end{aligned}$$

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TABLE 2. The edge label of a n-star-decomposition of G if m is odd and n is even.

f	v_1	v_2	v_3	...	v_{n-1}	v_n
u_{m-1}	$(m+n)$ $+1$	$(2m+n)$ $+m$	$(3m+n)$ $+1$	...	$((n-1)m+n)$ $+1$	$(n(m)+n)$ $+(\frac{m+3}{2})$
u_{m-3}	$(m+n)$ $+2$	$(2m+n)$ $+(m-1)$	$(3m+n)$ $+2$	...	$((n-1)m+n)$ $+2$	$(n(m)+n)$ $+(\frac{m+5}{2})$
u_{m-5}	$(m+n)$ $+3$	$(2m+n)$ $+(m-2)$	$(3m+n)$ $+3$	...	$((n-1)m+n)$ $+3$	$(n(m)+n)$ $+(\frac{m+7}{2})$
$\vdots$	...	...	...	...	...	...
u_k	$(m+n+1)$ $+(\frac{m-(k+1)}{2})$	$(3m+n)$ $-(\frac{m-(k+1)}{2})$	$(3m+n+1)$ $+(\frac{m-(k+1)}{2})$	...	$((n-1)m+n)$ $+1+(\frac{m-(k+1)}{2})$	$(n(m)+n)$ $+\frac{m+((m-k)+2)}{2}$
$\vdots$	...	...	...	...	...	...
u_2	$(m+n)$ $+(\frac{m-1}{2})$	$(2m+n)$ $+(\frac{m+3)}{2})$	$(3m+n)$ $+(\frac{m-1}{2})$	...	$((n-1)m+n)$ $+(\frac{m-1}{2})$	$(n(m)+n)$ $+n$
u_m	$(m+n)$ $+(\frac{m+1}{2})$	$(2m+n)$ $+(\frac{m+1)}{2})$	$(3m+n)$ $+(\frac{m+1}{2})$	...	$((n-1)m+n)$ $+(\frac{m+1}{2})$	$(n(m)+n)$ $+1$
$\vdots$	...	...	...	...	...	...
u_j	$(2m+n)$ $-(\frac{(j-1)}{2})$	$(2m+n+1)$ $+(\frac{(j-1)}{2})$	$(4m+n)$ $-(\frac{(j-1)}{2})$	...	$(n(m)+n)$ $-\frac{(j-1)}{2})$	$(n(m)+n)$ $+\frac{(m-(j-2))}{2}$
$\vdots$	...	...	...	...	...	...
u_3	$(2m+n)$ -1	$(2m+n)$ $+2$	$(4m+n)$ -1	...	$(n(m)+n)$ -1	$(n(m)+n)$ $+(\frac{m-1}{2})$
u_1	$(2m+n)$	$(2m+n)$ $+1$	$(4m+n)$	...	$(n(m)+n)$	$(n(m)+n)$ $+(\frac{m+1}{2})$

Now,

$$
\begin{aligned}
\sum_{i=1}^{n} f(u_j v_i) &= (2m+n-(\frac{j-1}{2})) + (2m+n+1+(\frac{j-1}{2})) \\
&\quad +(4m+n-(\frac{j-1}{2})) + (4m+n+1+(\frac{j-1}{2})) \\
&\quad +\cdots+(n(m)+n-(\frac{j-1}{2})) + (n(m)+n+\frac{(m-(j-2))}{2}) \\
&= ((2m+n)+(2m+n+1)+(4m+n)+(4m+n+1)+\cdots \\
&\quad +((n-2)m+n)+((n-2)m+n+1)) \\
&\quad +(n(m)+n-(\frac{j-1}{2})) + (n(m)+n+\frac{(m-(j-2))}{2}) \\
&= 2((2m+n)+(4m+n)+\cdots+((n-2)m+n)) + \frac{n-2}{2}(1) \\
&\quad +2(n(m)+n) + (\frac{(m-(j-2))}{2} - (\frac{j-1}{2})) \\
&= 2((2m+n)+(4m+n)+\cdots+((n-2)m+n)+(n(m)+n)) \\
&\quad +\frac{n-2}{2} + (\frac{(m-(j-2))}{2} - (\frac{j-1}{2})) \\
&= 2((2m+4m+\cdots+n(m)) + \frac{n}{2}(n)) + (\frac{m-2j+3}{2}) \\
&= 2(2m(1+2+\cdots+\frac{n}{2}) + \frac{n^2}{2}) + (\frac{m+n+3-2(j+1)}{2}).
\end{aligned}
$$

$$
\begin{aligned}
&= 2(2m(1+2+\cdots+\frac{n}{2})+\frac{n^2}{2})+(\frac{m+n+3-2(j+1)}{2})\\
&= 2(\frac{2m(n)(n+2)}{8}+\frac{n^2}{2})+\frac{m+n+3}{2}-(j+1)\\
&= 2(\frac{mn(n+2)+2n^2}{4})+\frac{m+n+3}{2}-(j+1)\\
&= (\frac{mn(n+2)+2n^2}{2})+\frac{m+n+3}{2}-(j+1)\\
&= \frac{mn^2+2mn+2n^2+m+n+3}{2}-(j+1)\\
&= \frac{mn^2+2mn+2n^2+m+n+3-1+1}{2}-(j+1)\\
&= \frac{(mn^2+2mn+m)+(2n^2+n-1)}{2}+\frac{4}{2}-(j+1)\\
&= \frac{m(n^2+2n+1)+(2n-1)(n+1)}{2}-(j-1)\\
&= \frac{m(n+1)^2+(2n-1)(n+1)}{2}-(j-1).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\sum f(H_j) &= j+mn+\frac{n(n+1)}{2}+\frac{m(n+1)^2+(2n-1)(n+1)}{2}-(j-1)\\
&= (mn+1)+\frac{(n+1)(n+m(n+1)+(2n-1))}{2}\\
&= (mn+1)+\frac{(n+1)(m(n+1)+(3n-1))}{2}\\
&= (mn+1)+\frac{(n+1)(n(m+3)+(m-1))}{2}.
\end{aligned}
$$

which is same as in Case 1. So in this case the graph G is a H-V-super-strong magic decomposable graph.

Subcase(ii): i is even, where $1 \leq i \leq m$.
We prove the result for $i = k$ and the result follows for all $1 \leq i \leq m$ and i is even.
From Table 2 and from definition of f, we get

$$
\begin{aligned}
\sum f(H_k) &= f(u_k)+\sum_{i=1}^{n} f(v_i)+\sum_{i=1}^{n} f(u_k v_i)\\
&= k+mn+\frac{n(n+1)}{2}+\sum_{i=1}^{n} f(u_k v_i).
\end{aligned}
$$

Now,

$$
\begin{aligned}
\sum_{i=1}^{n} f(u_k v_i) &= ((m+n)+1+\frac{(m-(k+1))}{2})+((3m+n)-\frac{(m-(k+1))}{2})\\
&\quad +((3m+n)+1+\frac{(m-(k+1))}{2})+\cdots\\
&\quad +(((n-1)m+n)+1-\frac{(m-(k+1))}{2})\\
&\quad +(n(m)+n+\frac{m+((m-k)+2)}{2}).
\end{aligned}
$$

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$$
\begin{aligned}
&= (m+n)+2((3m+n)+(5m+n)+\cdots+((n-1)m+n)) \\
&\quad +\frac{n}{2}(1)+\frac{(m-(k+1))}{2}+(n(m)+n)+\frac{m+((m-k)+2)}{2} \\
&= 2((3m+5m+\cdots+(n-1)m)+(\frac{n-2}{2})n)+(m+n)+\frac{n}{2} \\
&\quad +(n(m)+n)+\frac{m-k-1+m+m-k+2}{2} \\
&= 2(m(3+5+\cdots+(n-1))+\frac{n^2-n}{2})+nm+m+2n \\
&\quad +\frac{3m-2k+1+n}{2} \\
&= 2(m((1+2+\cdots+(n-1))-(2+4+\cdots+(n-2))-1)) \\
&\quad +nm+m+2n-n^2-2n+\frac{3m-2k+1+n+1-1}{2} \\
&= 2(m(\frac{(n-1)(n)}{2}-2(1+2+\cdots+\frac{n-2}{2})-1)) \\
&\quad +n^2+nm+m+\frac{3m+n-1}{2}-(k-1) \\
&= 2(m(\frac{n^2-n}{2}-2\frac{(n-2)(n)}{8}-1))+n^2+nm \\
&\quad +m+\frac{3m+n-1}{2}-(k-1) \\
&= 2(m(\frac{2n^2-2n-n^2+2n-4}{4}))+n^2+nm+m+\frac{3m+n-1}{2} \\
&\quad -(k-1) \\
&= 2(\frac{m(n^2-4)}{4})+\frac{2n^2+2nm+2m+3m+n-1}{2}-(k-1) \\
&= \frac{2n^2+2nm+5m+n-1+mn^2-4m}{2}-(k-1) \\
&= \frac{(mn^2+2nm+m)+(2n^2+n-1)}{2}-(k-1) \\
&= \frac{m(n+1)^2+(2n-1)(n+1)}{2}-(k-1).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\sum f(H_k) &= k+mn+\frac{n(n+1)}{2}+\frac{(n+1)(m(n+1)+(2n-1)}{2}-(k-1) \\
&= (mn+1)+\frac{(n+1)(n+m(n+1)+(2n-1))}{2} \\
&= (mn+1)+\frac{(n+1)(m(n+1)+(3n-1))}{2} \\
&= (mn+1)+\frac{(n+1)(n(m+3)+(m-1))}{2}.
\end{aligned}
$$

which is same as in Case 1. So in this case the graph G is a H-V-super-strong magic decomposable graph.

Case 3: m and n are odd.
Subcase(i): $m \neq n$. The table for this Case is exactly same as in Table 1 and hence we get the same result as in Case 1.

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TABLE 3. The edge label of a n-star-decomposition of G if n is odd.

f	v_1	v_2	...	v_{n-1}	v_n
u_1	$3n$	$3n+1$	...	$n(n)+1$	$(n+2)n$
u_2	$3n-1$	$3n+2$	...	$n(n)+2$	$(n+2)n-1$
u_3	$3n-2$	$3n+3$	...	$n(n)+3$	$(n+2)n-2$
$\vdots$	...	...	...	...	...
u_k	$3n-(k-1)$	$3n+k$	...	$n(n)+k$	$(n+2)n-(k-1)$
$\vdots$	...	...	...	...	...
u_{n-1}	$2n+2$	$4n-1$	...	$(n+1)(n)-1$	$(n+1)n+2$
u_n	$2n+1$	$4n$	...	$(n+1)(n)$	$(n+1)n+1$

Subcase(ii): $m=n$.
Now the edges of G can be labeled as shown in Table 3.
We prove the result for $n=k$ and the result follows for all $1 \leq k \leq n$.
From Table 3 and from definition of f, we get

$$\begin{aligned} \sum f(H_k) &= f(u_k)+\sum_{i=1}^{n} f(v_i)+\sum_{i=1}^{n} f(u_k v_i) \\ &= k+\sum_{i=1}^{n}(n+i)+\sum_{i=1}^{n} f(u_k v_i). \end{aligned}$$

Now,

$$\begin{aligned} \sum_{i=1}^{n} f(v_i) &= (n+1)+(n+2)+\cdots+(n+n) \\ &= n(n)+(1+2+\cdots+n) \\ &= n(n)+\frac{n(n+1)}{2}. \end{aligned}$$

Also

$$\begin{aligned} \sum_{i=1}^{n} f(u_k v_i) &= (3n-(k-1))+(3n+k)+(5n-(k-1))+(5n+k)+\cdots \\ &\quad +(n(n)-(k-1))+(n(n)+k)+((n+2)n-(k-1)) \\ &= 2n(3+5+\cdots+n)+\frac{n-1}{2}(1)+(n+2)n-(k-1) \\ &= 2n((1+2+\cdots+n)-(2+4+\cdots+(n-1))-1) \\ &\quad +\frac{2n^2+5n-1}{2}-(k-1) \\ &= 2n(\frac{n(n+1)}{2}-2(1+2+\cdots+(\frac{n-1}{2}))-1) \\ &\quad +\frac{2n^2+5n-1}{2}-(k-1) \\ &= 2n(\frac{n^2+n}{2}-\frac{(n-1)(n+1)}{4}-1)+\frac{2n^2+5n-1}{2}-(k-1). \end{aligned}$$

$$
\begin{aligned}
&= 2n(\frac{2n^2+2n-n^2+1-4}{4})+\frac{2n^2+5n-1}{2}-(k-1)\\
&= n(\frac{n^2+2n-3}{2})+\frac{2n^2+5n-1}{2}-(k-1)\\
&= \frac{n^3+4n^2+2n-1}{2}-(k-1)\\
&= \frac{(n^3+2n^2+n)+(2n^2+n-1)}{2}-(k-1)\\
&= \frac{n(n+1)^2+(2n-1)(n+1)}{2}-(k-1).
\end{aligned}
$$

Using the above values, we get

$$
\begin{aligned}
\sum f(H_k) &= k+n(n)+\frac{n(n+1)}{2}+\frac{n(n+1)^2+(2n-1)(n+1)}{2}-(k-1)\\
&= (n(n)+1)+\frac{(n+1)(n+n(n+1)+(2n-1))}{2}.\\
&= (n(n)+1)+\frac{(n+1)(n^2+4n-1)}{2}\\
&= (n(n)+1)+\frac{(n+1)(n^2+3n+(n-1))}{2}\\
&= (n(n)+1)+\frac{(n+1)(n(n+3)+(n-1))}{2}.
\end{aligned}
$$

Thus,

$$
\sum f(H_k) = (mn+1)+\frac{(n+1)(n(m+3)+(m-1))}{2}.
$$

Hence G is a H-V-super-strong magic decomposable graph. □

Theorem 3.2. *If a non-trivial H-decomposable graph $G \cong K_{m,n}$ is H-V-super-strong magic decomposable graph with at least m or n is odd and if the sum of edge labels of a decomposition H_j is denoted by $\sum f(E(H_j))$ then $\{\sum f(E(H_1)), \sum f(E(H_2)), \cdots, \sum f(E(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = \frac{m(n+1)^2+(2n-1)(n+1)}{2}$ and $d = -1$.*

Proof. Suppose G is H-decomposable and possesses a H-V-super-strong magic labeling f, then by Theorem 3.1, for each H_j in the H-decomposition of G, we get

$$
\begin{aligned}
\sum f(E(H_j)) &= \sum_{i=1}^{n} f(u_j v_i)\\
&= \frac{m(n+1)^2+(2n-1)(n+1)}{2}-(j-1).
\end{aligned}
$$

which is true for all $1 \le j \le m$. Thus $\{\sum f(E(H_1)), \sum f(E(H_2)), \cdots, \sum f(E(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = \frac{m(n+1)^2+(2n-1)(n+1)}{2}$ and $d = -1$. □

Theorem 3.3. *If a non-trivial H-decomposable graph $G \cong K_{m,n}$ is H-V-super-strong magic decomposable graph with at least m or n is odd and if the sum of vertex labels of a decomposition H_j is denoted by $\sum f(V(H_j))$ then $\{\sum f(V(H_1)), \sum f(V(H_2)), \cdots, \sum f(V(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = (mn+1)+\frac{n(n+1)}{2}$ and $d = 1$.*

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Proof. Suppose G is H-decomposable and possesses a H-V-super-strong magic labeling f, then by Theorem 3.1, for each H_j in the H-decomposition of G, we get

$$\begin{aligned}
\sum f(V(H_j)) &= f(u_j) + \sum_{i=1}^{n} f(v_i) \\
&= j + \sum_{i=1}^{n}(m+i) \\
&= j + ((m+1) + (m+2) + \cdots + (m+n)) \\
&= j + mn + \frac{n(n+1)}{2}.
\end{aligned}$$

which is true for all $1 \leq j \leq m$. Thus $\{\sum f(V(H_1)), \sum f(V(H_2)), \cdots, \sum f(V(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = (mn+1) + \frac{n(n+1)}{2}$ and $d = 1$. □

The following theorem gives a necessary and sufficient condition for a graph to be H-V-super-strong magic decomposable.

Theorem 3.4. *Let $G \cong K_{m,n}$ be a H-decomposable graph with at least m or n is odd and if $V(G) = U(G) \cup W(G)$ with $|U(G)| = m$ and $|W(G)| = n$. let g be a bijection from $V(G)$ onto $\{1, 2, \cdots, p\}$ with $g(U(G)) = \{1, 2, \cdots, m\}$ and $g(W(G)) = \{(m+1), (m+2), \cdots, (m+n = p)\}$ then g can be extended to an H-V-super-strong magic labeling if and only if $\{\sum f(V(H_1)), \sum f(V(H_2)), \cdots, \sum f(V(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = (mn+1) + \frac{n(n+1)}{2}$ and $d = 1$.*

Proof. Suppose $G \cong K_{m,n}$ be a H-decomposable graph with at least m or n is odd and if $V(G) = U(G) \cup W(G)$ with $|U(G)| = m$ and $|W(G)| = n$. let g be a bijection from $V(G)$ onto $\{1, 2, \cdots, p\}$ with $g(U(G)) = \{1, 2, \cdots, m\}$ and $g(W(G)) = \{(m+1), (m+2), \cdots, (m+n = p)\}$. Assume that $\{\sum f(V(H_1)), \sum f(V(H_2)), \cdots, \sum f(V(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = (mn+1) + \frac{n(n+1)}{2}$ and $d = 1$. Define $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ as $f(u_i) = g(u_i)$; $f(v_j) = g(v_j)$ for all $1 \leq i \leq m$; $1 \leq j \leq n$ and the edge labels are in either Table 1 (if m is even and n is odd (or) both m and n are odd, $m \neq n$) or Table 2 (if m is odd and n is even) or in Table 3 (if both m and n are odd, $m = n$) then by Theorem 3.1, for each H_j in the H-decomposition of G, we get

$$\begin{aligned}
\sum f(E(H_j)) &= \sum_{i=1}^{n} f(u_j v_i) \\
&= \frac{m(n+1)^2 + (2n-1)(n+1)}{2} - (j-1).
\end{aligned}$$

which is true for all $1 \leq j \leq m$. So $\{\sum f(E(H_1)), \sum f(E(H_2)), \cdots, \sum f(E(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = \frac{m(n+1)^2+(2n-1)(n+1)}{2}$ and $d = -1$. and hence, we have

$$\begin{aligned}
\sum f(H_j) &= \sum f(V(H_j)) + \sum f(E(H_j)) \\
&= (j + mn + \frac{n(n+1)}{2}) + (\frac{m(n+1)^2 + (2n-1)(n+1)}{2} - (j-1)) \\
&= ((mn+1) + \frac{n(n+1)}{2}) + (\frac{m(n+1)^2 + (2n-1)(n+1)}{2}) \\
&= (mn+1) + \frac{(n+1)(n(m+3) + (m-1))}{2}.
\end{aligned}$$

is constant for every H_j in the H-decomposition of G and for all $1 \leq j \leq m$. Thus we have, f is an H-V-super-strong magic labeling.

Suppose g can be extended to an H-V-super-strong magic labeling f of G with magic constant k.

Then $k = \sum f(H_j) = (mn+1) + \frac{(n+1)(n(m+3)+(m-1))}{2}$ for every H_j in the H-decomposition of G and for all $1 \leq j \leq m$. Since G is H-V-super-strong magic decomposable, by Theorem 3.3 we have $\{\sum f(V(H_1)), \sum f(V(H_2)), \cdots, \sum f(V(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = (mn+1) + \frac{n(n+1)}{2}$ and $d = 1$. $\square$

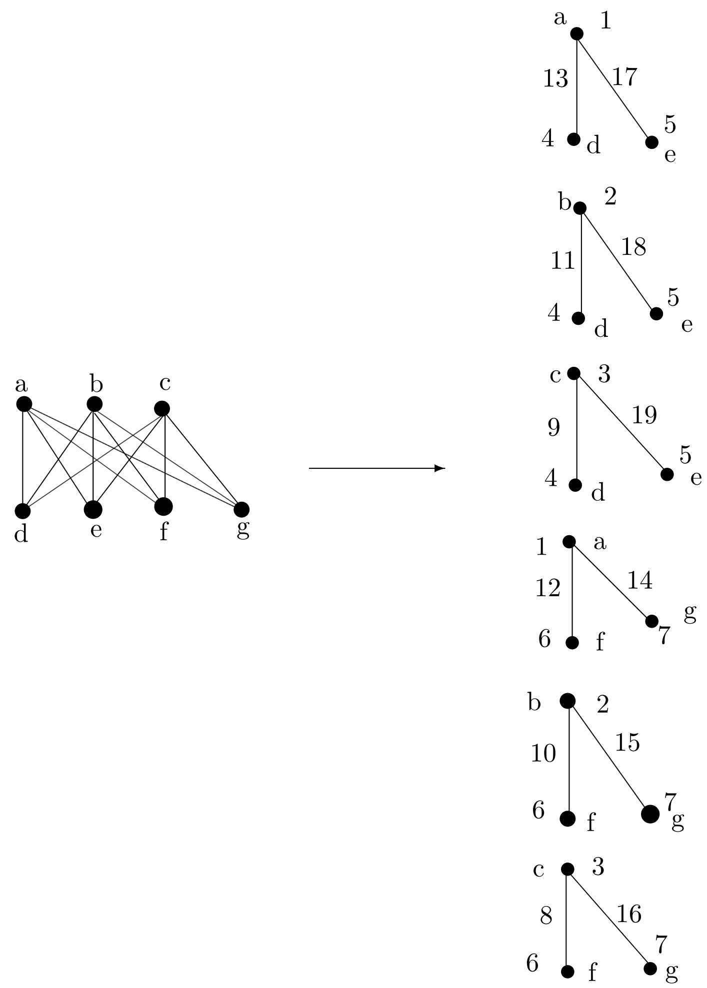

FIGURE 1. 2-star-V-super-strong magic decompositon of $K_{3,4}$

The following theorem is useful in finding classes of graphs that are not H-V-super strong magic.

Theorem 3.5. *Suppose $\{H_1, H_2, \cdots, H_m\}$ is a H-decomposition of $G \cong K_{m,n}$ with both m and n are even. Then G is not a H-V-super-strong magic decomposable graph.*

Proof. Suppose G is H-V-super-strong magic decomposable, then by Theorem 3.2, we have $\{\sum f(E(H_1)), \sum f(E(H_2)), \cdots, \sum f(E(H_m))\} = \{a, a+d, \cdots, a+(m-1)d\}$ with $a = \frac{m(n+1)^2+(2n-1)(n+1)}{2}$ and $d = -1$. It is given than both m and n are even. We take $m = 2s$ and $n = 2t$. Therefore

$$a = \frac{2s(2t+1)^2 + (2(2t)-1)(2t+1)}{2}.$$

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$$= \frac{2s(4t^2+4t+1)+(8t^2+2t-1)}{2}$$
$$= \frac{2(4st^2+4st+4t^2+s+t)-1}{2}.$$

which is not an integer and hence $G \cong K_{m,n}$ is not a H-V-super-strong magic decomposable graph if both m and n are even. □

4. Conclusion

In this paper, we have given a mathematical modeling of campus information system which helps us to define H-V-super strong magic labeling. Also, we have given a complete characterization of n-star-V-super-strong magic decomposition of $K_{m,n}$, $n \geq 1$ and with at least m or n is odd.

Further more, Figure 1 shows that $K_{3,4}$ is a 2-star-V-super-strong magic decomposable graph with magic constant $k = 40$. Let $U = \{a, b, c\}$ and $W = \{d, e, f, g\}$ be two stable sets of $K_{3,4}$ such that $V(G) = U \cup W$. Let $\{H_1 = \{(a, d), (a, e)\}, H_2 = \{(b, d), (b, e)\}, H_3 = \{(c, d), (c, e)\}, H_4 = \{(a, f), (a, g)\}, H_5 = \{(b, f), (b, g)\}, H_6 = \{(c, f), (c, g)\}\}$ be a H-decomposition of $K_{3,4}$, where each H_i is isomorphic to $H \cong K_{1,2}$, for all $1 \leq i \leq 6$.

It is natural to have the following problem.

Open Problem 4.1. *Discuss the m-star-V-super-strong magic decomposition of $K_{m,n}$ with $1 \leq m < n$.*

References

[1] J. Akiyama and M. Kano, *Path factors of a graph*, Graphs and Applications, Wiley, Newyork, (1984).
[2] Andrew S. Tanenbaum, David J. Wetherall, *Computer Networks*, 5th Edition, Prentice Hall (2013).
[3] G. Chartrand, L. Lesniak, *Graphs and Digraphs*, 3rd Edition, Chapman and Hall, Boca Raton, London, Newyork, Washington, D.C (1996).
[4] G. Chartrand and P. Zhang, *Chromatic Graph Theory*, Chapman and Hall, CRC, Boca Raton, (2009).
[5] H. Emonoto, Anna S Lladó, T. Nakamigawa and G. Ringel, Super edge-magic graphs, *SUT J. Math.* **34** (1998) 105-109.
[6] Forouzan, *Data Communication and Networking*, 5th Edition, Tata McGraw Hill (2014).
[7] J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.* **17** (2014) #*DS*6.
[8] J. A. MacDougall, M. Miller, Slamin, W.D. Wallis, Vertex-magic total labelings of graphs, *Util. Math.* **61** (2002) 3-21.
[9] J. A. MacDougall, M. Miller and K. Sugeng, Super vertex-magic total labeling of graphs, *Proc. 15th AWOCA* (2004) 222-229.
[10] G. Marimuthu, M. Balakrishnan, E-super vertex magic labelings of graphs, *Discrete Appl. Math.* **160** (2012) 1766-1774.
[11] G. Marimuthu, M. Balakrishnan, Super edge magic graceful graphs, *Inform. Sci.* **287** (2014) 140-151.
[12] A. M. Marr, W. D. Wallis, *Magic graphs*, Birkhauser, Boston, Basel, Berlin, (2013) (2nd edition).
[13] F. A. Muntaner-Batle, Special super edge magic labelings of bipartite graphs, *J.combin. Math. Combin. Comput.* **39** (2001), 107-120
[14] J. Sedlacek, Problem 27, *Theory of Graphs and its Applications*, Proceedings of Symposium Smolenice, (1963), pp. 163-167.
[15] S. Stalin Kumar, G. Marimuthu, H-V-super magic decomposition of complete bipartite graphs, *Commun. Korean Math. Soc.* **30** (2015) 313-325.
[16] S. P. Subbiah, J. Pandimadevi, H-E-Super magic decomposition of graphs, *Electronic Journal of Graph Theory and Applications* **2** (2) (2014) 115-128.
[17] K. A. Sugeng and W. Xie, Construction of Super edge magic total graphs, *Proc. 16th AWOCA* (2005), 303-310.

[18] Yoshimi Egawa, Masatsugu Urabe, Toshihito Fukuda and Seiichiro Nagoya, A Decomposition of Complete bipartite graphs into edge-disjoint subgraphs with star components, *Discrete Math.* **58** (1986) 93-95.

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